AN ANALYSIS OF THE SEPARATION OF COAL INTO LOW-DENSITY AND HIGH-DENSITY FRACTIONS ON A CONCENTRATING TABLE

BY

CHARLES CHILTON BOLEY

B.S., University of Illinois, 1935
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DEPARTMENT OF REGISTRATION AND EDUCATION
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ANALYSIS OF COAL CLEANING ON A CONCENTRATING TABLE

BY

CHARLES C. BOLEY

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URBANA, ILLINOIS
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Topographic Mapping in Cooperation with the United States Geological Survey.

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November 15, 1948

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ANALYSIS OF COAL CLEANING ON A CONCENTRATING TABLE

BY

CHARLES C. BOLEY

INTRODUCTION

THE STOKER COAL testing program of the Illinois State Geological Survey included a need for equipment to improve coals, so that the effect on stoker combustion of variations in chemical and petrographic characteristics in the same lot of coal could be explored. It was desired that the equipment to produce these changes be readily adaptable to any Illinois coal; that it be of a continuous-flow rather than batch type, so as to simulate plant operation; and that it be capable of making an empty start and coming to equilibrium operating conditions without undue consumption of coal or time.

These specifications are admirably met by the concentrating table, on which separation of coal from impurity takes place in full view in a comparatively wide and shallow bed. The concentrating table is limited to a rather small maximum particle size, but experimental interest centered around stoker-size coal so that the concentrating table was judged to be well suited to the needs of the coal testing program.

When the work began the extent of the stoker coal testing program was not known. It was anticipated that many tabling runs would be involved and that flexible control of the tabling operations might yield data both interesting and of potential use in coal preparation, relating the operation of the table to the coals washed and the results obtained.

ACKNOWLEDGMENTS

All chemical analyses were made by the Analytical Division of the Illinois State Geological Survey under the supervision of O. W. Rees, Head of the Division. The majority of the washability analyses and some of the size analyses were made by Otto I. Godoy, a cooperating student in the Department of Mining and Metallurgical Engineering, of the University of Illinois.

Roy J. Helfinstine, Mechanical Engineer, Coal Division of the Survey, was closely associated in all phases of the investigation, and his helpful advice is gratefully acknowledged.

Everett L. Welker, Professor, Department of Mathematics, University of Illinois, kindly assisted in planning the treatment of the data. Robert Root and O. F. Smith, Technical Assistants in the Coal Division of the Illinois State Geological Survey, helped handle the large quantities of coal.

A special expression of gratitude for their unfailing interest and encouragement is due Gilbert H. Cady, Senior Geologist and Head of the Coal Division of the Illinois State Geological Survey, who made the work possible, and to H. L. Walker, Head of the Department of Mining and Metallurgical Engineering of the University of Illinois, who proposed the cooperative program and under whom this thesis was prepared in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois in 1947.

Most of the coal samples were contributed by individual coal companies, whose cooperation is sincerely appreciated.

OBJECTIVES

The major objectives of the work were the collection of comparable data on the cleaning of a number of Illinois coals on a concentrating table and the study of the

influence of the chief operating variables on tabling results.

It was clear from the start that these objectives, to be fully met in a strictly experimental manner, would require a more extensive work schedule than could be fitted into the regular laboratory program. Ideally, it would involve testing a relatively large number of samples of a single coal, varying each variable in turn over some reasonable range while maintaining the other variables substantially constant, followed by similar programs on each of several other typical coals. The time requirements and expense of such work, where each test is on pilot-plant scale, were judged to be impracticably large.

However, the established laboratory program called for a number of Illinois coals to be tabled in connection with the combustion phase of the investigation, and it was thought that the fund of data which could be secured from this work would permit statistical analysis for the type of information desired on tabling operation.

A secondary objective was the study of the economic justification and advantages of coal cleaning, with particular reference to the concentrating table.

SCOPE

Twelve samples of Illinois coal, representing all important mining districts, were cleaned on a laboratory concentrating table under conditions of special control, two runs being made on most coals. Five of the cleaned coals and two of the lots of material rejected from the initial tablings were retabled.

The operating variables regarded as most important and arranged to be controlled were transverse slope, longitudinal slope, length of stroke, frequency of stroke, rate of coal feed, and rate of introduction of wash water. Variables inherent in table design—shape of deck, system of riffling, and type of reciprocating motion—were held constant by using the same table throughout the work.

All samples were prepared to the same size range before tabling. Complete operating data for the tabling runs were obtained, and all products were chemically analyzed. Size analyses were run on all head samples and clean-coal products, excepting for the two lots of rejected material that were retabled.

A study was made of the fractionation of high bed-moisture coals by specific gravity methods; a procedure more reliable but somewhat more complex than usual was adopted; and washability data were obtained for all but one of the raw coals.

The influence of each of the several major operating variables on tabling performance is analyzed.

Probable effects on cost, convenience, and coal performance that are assignable to coal cleaning are analyzed from the standpoint of the domestic consumer of stoker coal, and the merits of the concentrating table as a coal-cleaning device are discussed.

THEORY OF TABLING

A concentrating table is a development of the ancient principle of flowing-film concentration, refined by the addition of various devices to improve and to make continuous its performance. In common with most coal-cleaning processes, tabling takes advantage of differences in specific gravity between coal and its associated impurities, all of which are more dense than coal.

The table is essentially an almost horizontal deck, rectangular in shape and reciprocated in the direction of its long axis by a suitable mechanism (usually a toggle and pitman). The mechanism causes an asymmetrical acceleration of the deck, such that particles on it move intermittently toward one end. Numerous parallel cleats, or riffles, are applied to the deck in a direction essentially parallel to its reciprocation, although with many variations as to height, length, spacing, taper, and direction. During operation the deck is tilted a few degrees in a direction perpendicular to its reciprocation, a sheet of water is allowed to flow across it, and coal is fed at the upper corner farthest from the discharge end. The motion of any particle in the feed across the deck is the resultant of the force imposed

by the longitudinal motion of the deck and that imposed by the transverse flow of water. The effects these forces have on particles differing in size and specific gravity are outlined in more detail in succeeding paragraphs, but in general it may be said that particles of higher specific gravity are affected more by the motion of the deck and hence tend to move to the end of the deck; whereas particles of lower specific gravity are affected more by the cross-flow of water and hence tend to move to the lower side of the deck.

The classical theory is somewhat more detailed.[1] It holds that as the feed material is subjected to the joint action of deck motion and water flow, it fans out from the feed corner and builds up in layers behind the riffles. Here it is delayed momentarily as a loose bed of solids which are buoyed up by water and free to move to a certain extent relative to each other, and a limited degree of stratification takes place. The smaller particles move downward, the heavier more promptly than the lighter, while the larger and lighter particles move up, where they are exposed to the transverse flow of water and are washed over the top of the riffle.[2] This cycle of partial separation is repeated over each riffle, each step being "in itself inefficient, but by virtue of the numerous retreatments valuable results are obtained."[3]

Meantime the material trapped behind the riffle is moved by the differential reciprocation of the deck toward the discharge end of the table. The riffles, which taper downward in height toward the discharge end, permit the progressive removal of successively deeper layers of material by the cross-flow of water. The material carried over the riffle is subjected to further retreatments as it encounters further riffles. Thus heavy material tends to be diverted longitudinally while light material is washed laterally by the cross currents of water.

A particle finally reaches the coal-discharge edge only if it is of low enough specific gravity to climb every riffle. Particles of high specific gravity may climb some of the riffles; but when the table is in proper adjustment, they will be carried by the reciprocating motion to the refuse discharge end. Particles of intermediate specific gravity are detained longer behind the riffles, climbing them only after having moved downstream where the riffles are lower.

However, modern opinion is that this theory of stratification and exposure of successively deeper layers of strata to the cross-flow of water does not adequately account for the highly efficient separations that tables are known to be capable of making.[4] In particular, it does not explain the presence of fine material of low specific gravity which passes over the coal-discharge edge of the table long before the taper of the riffles would expose it to the direct action of the cross-flowing currents of water.[5]

Bird and Davis[6] devised special apparatus to explore the effects of pure stratification, with complete elimination of cross-flowing water and of differential deck motion. Their tests, although not exhaustive, appear to demonstrate rather clearly that stratification alone cannot be credited with the separations which take place.

Bird and Davis suggest that there may be a certain amount of hindered settling between the riffles, as a consequence of that portion of the cross-flowing water which flows through the interstices of the bed of particles, rather than over the top. The normal action of stratification would cause the interstices toward the bottom of the bed to be smaller, which, in addition to the effect of skin friction between the water and the deck, would be expected to cause progressively slower water currents in progressively lower strata. Thus the velocity of the water roughly matches the size of the particles in the different strata; and a rather complex hindered settling takes place, hori-

[1] Gandrud, B. W. Concentrating tables. Chapter 13, pp. 425–56, of Coal preparation, David R. Mitchell, Editor; AIME, 729 pp., 1943; pp. 433–4.

[2] The term "stratification" as here employed corresponds to the term "consolidation trickling" as used by A. M. Gaudin in Chapter XII of "Principles of Mineral dressing." McGraw-Hill (New York), 554 pp., 1939.

[3] Thomas, B. D. Principles of gravity concentration. Chapter 9, pp. 249–73, of Coal preparation, David R. Mitchell, Editor; AIME, 729 pp., 1943; p. 265.

[4] Gandrud, B. W. Op. cit., p. 434.

[5] Bird, B. M. and Davis, H. S. The role of stratification in the separation of coal and refuse on a coal-washing table. U. S. Bur. Mines RI 2950, 19 pp., 1929; p. 18.

[6] Bird, B. M., and Davis, H. S. Op. cit.

zontal in part and veering to vertical as the next riffle is approached. This analysis is essentially an amplification of the effects ascribed by Taggart[7] to eddying between the riffles.

Such hindered settling as may take place between any two riffles would be aided by that taking place between succeeding riffles, and the net effect across an entire table might well be of considerable magnitude. The combination of stratification and hindered settling in this way could effect a net separation almost entirely assignable to specific gravity. In accordance with this theory, stratification brings light coarse material to the top of the bed at once, where it is promptly carried to the coal-discharge edge by the cross-flow of water. Light fine material, which pure stratification would deposit in the lower strata of the bed, is preferentially carried horizontally between the riffles and assisted over the riffles in a type of hindered settling by the cross-flowing currents of water.

Gaudin's analysis[8] of the interactions taking place in tabling places less emphasis upon the importance of riffles in causing a separation essentially on the basis of specific gravity. On the basis of reasonable assumptions, he shows that the direction of motion of a particle on a bare-decked table is a function almost entirely of its specific gravity, with little effect due to size; while its net amplitude or rate of motion is roughly proportional to the square of its diameter.

His analysis of the forces involved is particularly appealing from the theoretical standpoint, in that he attempts to establish functional relationships by proceeding from idealized conditions step by step toward actual tabling conditions. Thus he first considers a flowing film, and relates velocity, depth, and total volume flowing on the basis of the physics involved. This is followed by the development of the equation of motion of a single particle at the bottom of a flowing film; next are considered the forces acting on a particle in an ideal nonviscous liquid on a horizontal deck, horizontally moving with asymmetrical acceleration. Under the last-named conditions, which are practically approximated by a large particle in a deep film of water, a lower acceleration suffices to cause motion in a particle of lower specific gravity. Size of particle does not enter into the relationship. But when account is taken of fluid resistance, it becomes probable that net rate of motion also varies as some power of the size, probably between 1 and 2. This situation is theoretically very difficult and has not yet been satisfactorily analyzed.

Riffles on a deck increase capacity tremendously, converting the concentrating table into a practicable device. However, they introduce the phenomena of hindered settling and stratification (consolidation trickling) between each pair of riffles. In accordance with these principles, the smaller and heavier particles work to the bottom and the larger and lighter to the top. For a set of conditions approximating those of a table, the maximum velocity of water caused by deck motion at a point one millimeter above the deck is shown to be only about two percent of that of the deck.[9] Since the effect of lengthwise motion of the deck is felt almost solely by particles resting directly on it, the smaller and heavier particles move much more rapidly longitudinally than the larger and lighter.

It will be noted that this situation is exactly the opposite of that deduced for an unriffled deck, with a bed only one particle deep, that is, with all particles in contact with the deck. Fortunately, the resulting mixture of fine-light and coarse-heavy particles is the reverse of the type of mixture produced by pure classification, making classification of feed prior to tabling technically desirable, as has been pointed out.[10] However, other evidence, primarily in mineral dressing technology, indicates that classifying before tabling may be little if any

[7] Taggart, Arthur F. Handbook of ore dressing. Wiley (New York), 1679 pp., 1927; p. 719.

[8] Gaudin, A. M. Principles of mineral dressing. McGraw-Hill (New York), 554 pp., 1939; Chapter XIII, Flowing-film concentration and tabling.

[9] Gaudin, A. M. Op. cit.; table 38, p. 297.

[10] Richards, Robert H. The Wilfley table, I. Trans. AIME Vol. 38, pp. 556–80, 1907.
Bird, Byron M. The sizing action of a coal-washing table. U. S. Bur. Mines RI 2755, 8 pp., 1926.
Bird, B. M. and Yancey, H. F. Hindered-settling classification of feed to coal-washing tables. Trans. AIME Vol. 88, pp. 250–71, 1930.

superior to sizing before tabling. Practice seems to favor classifying before tabling in mineral dressing,[11] probably owing in part to the fact that classifying is easier than close sizing when handling fine material, while in coal preparation sizing before tabling is more common. As a matter of fact at least one authority states that tabling an unsized coal often produces the best results.[12]

In any event, it follows from Gaudin's analysis that there should be a nearly pure specific gravity separation on a bare deck under optimum conditions with a bed only one particle deep, and that riffling is actually a detriment to separation as it introduces a sizing action. But the relative capacity of a riffled deck is so much greater than that of an unriffled deck that bare decks are uncommon in mineral dressing and unknown in coal cleaning.

Gaudin's analysis, developed largely by reasoning from idealized conditions and free from detailed case histories, may not impress an operator as having much value, yet such thoughtful dissections of complex phenomena permit the clearest understanding of the forces involved and may suggest principles on which to base practical improvements, whereas full-scale experimentation may be unrevealing. Although much testing has been done on tabling and many data assembled, the conclusions in many cases may be of value only for the particular table or riffling system employed. The work reported herein is unquestionably open to this criticism. Gandrud is probably correct in stating, "As far as is known, no exhaustive studies have ever been made of the principles involved in table concentration by either ore-dressing or coal-preparation engineers."[13]

REVIEW OF PREVIOUS WORK

Many thousands of tons of coal have been tabled at hundreds of operating plants over the years, and out of this experience a great many reports of operating data have been made. Very few, however, have attempted to analyze the tabling process; and only one is known which considers the effect on separation of the major operating variables, the primary objective of this study. Moreover, no report on tabling has been seen which concerns itself with varying apparent specific gravity of coal particles as a function of moisture.

The Northwest Experiment Station of the United States Bureau of Mines has contributed the results of careful investigations on coal tabling.[14, 15, 16, 17, 18] Sizing action was studied, and the desirability of a classified feed (i.e., a feed in which coarse-light and fine-heavy material are grouped together) was demonstrated.[14, 15] For most coals, differential effects due to shape of particles are in the direction of improved performance, because flat or flaky material tends to be discharged from the table farther from the head-motion end than cubical material of the same specific gravity and screen size.[17] Inasmuch as impurities tend to be more tabular than coal, separation is aided rather than hindered.

A special device was constructed to study pure stratification, free from such other factors as differential table motion and cross flow of water.[15] Tests demonstrated fairly conclusively that stratification alone will not bring about the excellent separations of which concentrating tables are capable. The authors suggest the possibility that hindered settling between the riffles may contribute to the separating effect.

Continuing its work on coal tabling, the Northwest Experiment Station studied the effect of certain operating variables on efficiency of separation.[16] The objectives of this investigation were to establish the relationship which rate of deck movement, distribution of coal on the deck, and rate of coal feed have to efficiency of separation.

[11] Taggart, Arthur F. Op. cit., p. 758.
[12] Stone, S. A. (Deister Concentrator Company). Letter of Feb. 12, 1942, to the author.
[13] Gandrud, B. W. Op. cit., p. 435.
[14] Bird, Byron M. Op. cit.
[15] Bird, B. M. and Davis, H. S. Op. cit.
[16] Yancey, H. F., and Black, C. G. The effect of certain operating variables on the efficiency of the coal-washing table. U. S. Bur. Mines RI 3111, 13 pp., 1931.
[17] Yancey, H. F. Determination of shapes of particles and their influence on treatment of coal on tables. Trans. AIME Vol. 94, pp. 365–68, 1931 (TP 341).
[18] Bird, B. M., and Yancey, H. F. Hindered-settling classification of feed to coal washing tables. Trans. AIME Vol. 88, pp. 250–71, 1930 (TP 76).

Distribution of coal on the deck was evaluated by the percentage of feed discharged in the 4-foot zone of the coal-discharge edge nearest the head motion. Zonal samples of all discharged products were taken and individually analyzed for ash, permitting calculation of yield-ash performance by tabling for comparison with theoretical yield-ash data from a specific-gravity analysis. Efficiency of separation was computed for each of several washed coal ash contents by comparing actual yield of washed coal of any selected ash content with the theoretically possible yield of coal of the same ash content, as read from the table yield-ash curve and the specific gravity analysis.

Using a full-sized concentrating table and one selected coal of approximately 3-mesh by zero size, 33 tests were run. Rate of deck movement was varied at several levels of rate of coal feed, adjusting all other variables as needed to give best visual operation; and distribution was varied at several levels of rate of coal feed with constant deck movement, adjusting other variables as needed for best visual operation.

It was concluded that increased rate of deck movement, within the range explored, was conducive to increased efficiency of separation; that distribution, as measured in the indicated manner, had an optimum value above or below which results were inferior; and that efficiency decreased with increase in rate of coal feed.

EXPERIMENTAL WORK

Equipment

The coal washing unit available for the investigation was a laboratory-size concentrating table, equipped with a diagonal linoleum-covered deck and wooden riffles. The dimensions of the deck were approximately 8′8″ by 4′7″ (figure 1). The riffling system was known as "uphill" (riffles inclined at a slight angle to the line of motion of the table, carrying particles uphill against the flow of water) and was recommended by representatives of the manufacturer as being the nearest approach to a universal system and as most used in their own laboratory. Asymmetrical reciprocation of the table was caused by a toggle and pitman mechanism, standard for the full-size table; and a trough with adjustable openings across the upper edge of the table permitted control of the flow of water across the table.

Special effort was made to permit independent adjustment of each of the six operating variables (p. 8) without interrupting operation. The ability to make adjustments during operation was of special importance because it permitted close shadings of adjustment while observing the operation and

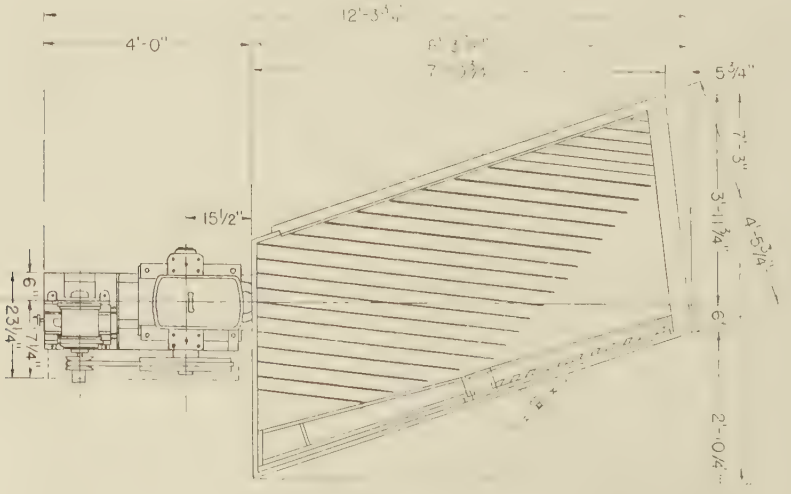

FIG. 1.—Concentrating table and drive.

made possible great savings in time and coal. Provision for adjusting the transverse slope during operation was incorporated in the table as purchased. In the present installation, the construction was modified to permit adjustment during operation of the longitudinal slope also. An infinitely variable speed changer permitted adjustment of speed of reciprocation, normally of the order of 270 strokes per minute. Length of stroke could be adjusted during operation up to a maximum of about 1½ inches.

Coal was fed to the table from a bin of approximately 3500 pounds capacity by a vibrating feeder, controlled by a variable voltage auto transformer, which could be calibrated for rate of coal flow. Water flow was metered, and a calibrated manometer indicated rate of flow.

Since the combustion phase of the program of which this work was a part required at least 1500 pounds of coal produced under stable washing conditions, and since relatively small quantities of coal were available for the entire procedure of establishing equilibrium from an empty start and of carrying on the washing, a recirculating system was developed,[19] consisting of a flight conveyor, a bucket elevator, and appropriate launders and chutes (figure 2). By means of this equipment it was possible to draw off a relatively small (200 to 300 lb.) quantity of coal from the feed bin and to experiment at length, without further consumption of coal, in order to establish desired working conditions. During the period of recirculation, coal and non-coal particles were separated on the table, recombined by launders, and dropped into relatively quiet water in a large tank where they settled into the trough of the flight conveyor. Water used in the tabling process was allowed to overflow the tank, under conditions such that very little coal was lost with the water.

The recombined material was then conveyed up a dewatering section and dropped into the boot of a bucket elevator, from which it was elevated to a point which permitted chuting it back to the feed box of the table.

A simple redisposition of two deflectors permitted the continuous withdrawal of the separated products, when desired washing conditions had been established by recirculation of the sample.

Sampling boxes with compartments were used for taking samples of the table products at various points along the discharging edge and for rapid estimation of percentage of reject being produced at any time.

A three-surface vibrating screen, accommodating wire-mesh screening surfaces 17 by 32 inches in size, was used for screening operations, and a small jaw crusher and a 12 by 10 inch smooth-surface double-roll crusher were used for crushing. Standard riffling and size-testing equipment was also available.

Coal Samples

Samples of from four to five tons were obtained from each of twelve shaft mines distributed throughout the major coal field in Illinois (table 1 and figure 3). Coals classified as of high volatile bituminous A, B, and C rank[20] were represented. All samples were unwashed and without surface treatment, and all but two were screenings or dedusted screenings. The two exceptions were run-of-mine coal from small operations.

Complete proximate and ultimate chemical analyses appear in table 2.

Procedure

TABLING

Preliminary experimentation and work by others[21] had indicated that the concentrating table available for use would not effectively handle coal particles over ¾-inch. When the top size was restricted to ½-inch, results were generally satisfactory. It is well known that a relatively narrow size range permits a more nearly true specific

[19] "Provision for continuous circulation of feed during the period while the table is being adjusted . . . is an especially valuable feature and should be incorporated in coal-washing test plants of any type." Yancey, H. F., and Fraser, Thomas. Coal washing investigations. U. S. Bur. Mines Bull. 300, 259 pp., 1929; p. 72.

[20] Standard specifications for classification of coals by rank. Amer. Soc. for Testing Materials, Designation D 388–38, 6 pp., 1938.

[21] Olin, H. L. The preparation of stoker coals from Iowa screenings. Univ. Iowa, Studies in Eng. Bull 28, 60 pp., 1942.

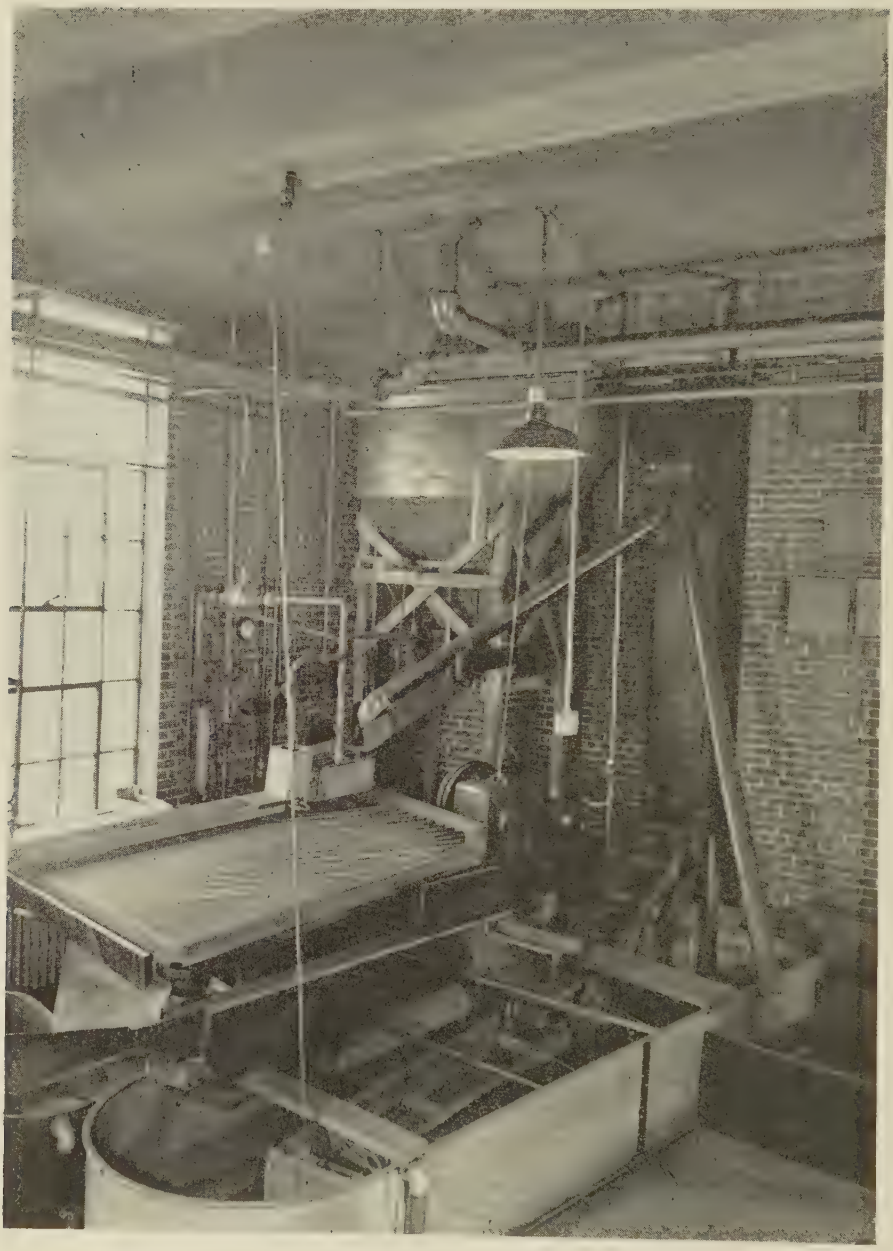

FIG. 2.—Concentrating table and auxiliary equipment.

gravity separation, and the size range was accordingly standardized at ½-inch by 8-mesh. This size corresponds very closely to a popular Illinois stoker coal used for earlier work. Removal of the minus 8-mesh fine material also minimized the loss of coal as slurry during washing, provided a cleaner handling coal, and minimized segregation during handling. The small top size served to minimize sampling errors.

For the screening work, ½-inch, 4-mesh and 8-mesh screening surfaces were used in the laboratory vibrating screen. The 4-mesh surface acted to relieve the 8-mesh

TABLE 1. SOURCE AND DESCRIPTION OF SAMPLES

Coal No.	County	Coal bed	Mining district[a]	County rank index[b]	Rank High volatile bituminous A, B, or C	Description of coal
5	Macoupin	Herrin No. 6	Central Illinois	121	C	1¼″ screenings
6	Peoria	Springfield No. 5	Fulton-Peoria	122	C	1¼″ screenings
7	Gallatin	Harrisburg No. 5	Eagle Valley	145	A	Mine run
8	Wabash	Friendsville	Mt. Carmel	122	C	Largely 2″ or 3″ screenings with some lump
9	St. Clair	Herrin No. 6	Belleville	126	C	Mine run. Considerable fine coal included
10	Saline	Harrisburg No. 5	Saline	137	B	Stoker, 1½″ by 10 mesh
11	Vermilion	Danville No. 7	Danville	125	C	Small stoker, 1″ by 8 mesh
12	Sangamon	Springfield No. 5	Springfield	121	C	Largely stoker, 1″ by 5/16″, with some 5/16″ by 0
13	Randolph	Herrin No. 6	Southwestern Illinois	126	C	Crushed 1½″ by ¾″
14	Christian	Herrin No. 6	Central Illinois	123	C	1½″ screenings
15	Williamson	Herrin No. 6	Franklin-Williamson	133	B	1½″ dedusted screenings
16	Knox	Rock Island No. 1	Northwestern Illinois	123	C	Stoker, 1″ by ¼″

[a] Designation of mining districts approximately as used in: Bement, A., Illinois coal: Illinois Geol. Survey Bull. 56, p. 23, 1929.
[b] Average heating value, expressed in hundreds of B.t.u. per pound on a moist mineral-matter-free basis, for face samples reported up to October, 1934. See Cady, Gilbert H., Classification and selection of Illinois coals: Illinois Geol. Survey Bull. 62, 354 pp., 1935.

FIG. 3.—Location of samples.

surface, permitting it to screen more effectively. Coal passing over the ½-inch screen surface was crushed and rescreened, this being repeated if necessary. Minus 8-mesh particles were sampled, weighed, and discarded.

For a tabling run, the feed bin was filled, the table was put into motion, wash water was introduced, and small increments of coal were allowed to flow onto the deck. With the recirculating system in operation, the table products were recombined, drained of water, elevated, and re-introduced to the table in a continuous process. However, conditions of separation were unstable until enough material had entered the system to build up a bed on the deck of the table and to fill certain points in the system, notably

TABLE 2.—ANAL-

Sample No.	5	6	7	8
SOURCE				
County	Macoupin	Peoria	Gallatin	Wabash
Coal seam number	6	5	5	[a]
LABORATORY NUMBER	C-2697	C-2776	C-2912	C-2932
PROXIMATE ANALYSIS				
As received basis:				
Moisture, percent	12.6	14.3	2.8	10.6
Ash, percent	16.6	15.5	13.9	20.8
Volatile matter, percent	34.3	29.9	33.9	34.0
Fixed carbon, percent	36.5	40.3	49.4	34.6
Moisture-free basis:				
Ash, percent	19.0	18.1	14.3	23.3
Volatile matter, percent	39.2	34.9	34.9	38.1
Fixed carbon, percent	41.8	47.0	50.8	38.6
Moisture- and ash-free basis:				
Volatile matter, percent	48.4	42.7	40.7	49.6
Fixed carbon, percent	51.6	57.3	59.3	50.4
ULTIMATE ANALYSIS				
As received basis:				
Hydrogen, percent	5.48	5.61	5.07	5.16
Carbon, percent	54.76	56.20	68.85	54.45
Nitrogen, percent	0.97	1.04	1.52	1.39
Oxygen, percent	17.40	18.04	5.41	15.42
Sulfur, percent	4.79	3.54	5.21	2.77
Ash, percent	16.60	15.57	13.94	20.81
Moisture-free basis:				
Hydrogen, percent	4.67	4.69	4.89	4.46
Carbon, percent	62.64	65.55	70.82	60.87
Nitrogen, percent	1.11	1.21	1.56	1.55
Oxygen, percent	7.12	6.26	3.04	6.76
Sulfur, percent	5.47	4.13	5.35	3.09
Ash, percent	18.99	18.16	14.34	23.27
Moisture- and ash-free basis:				
Hydrogen, percent	5.76	5.73	5.70	5.81
Carbon, percent	77.32	80.06	82.64	79.33
Nitrogen, percent	1.37	1.48	1.82	2.02
Oxygen, percent	8.79	7.68	3.59	8.81
Sulfur, percent	6.76	5.05	6.25	4.03
HEATING VALUE				
As received basis, B.t.u./lb	9955	10095	12297	9655
Moisture-free basis, B.t.u./lb	11387	11776	12649	10794
Moisture- and ash-free basis, B.t.u./lb	14058	14383	14760	14067
ASH FUSION TEMPERATURES				
Initial deformation, deg. F	1996	1960	1927	2065
Softening point, deg. F	2048	2002	2002	2113
Fluidity, deg. F	2346	2215	2155	2618

[a] Friendsville

the boot of the bucket elevator. When this stage was reached, the adjustments of the table could be varied in any way considered necessary and for as long a period as desired to secure a visually satisfactory separation.

When the separation was visually satisfactory, the reject and the cleaned coal were diverted out of the system, and simultaneously the feeder from the main bin was started, sending a steady flow of raw coal to the table. Thus the adjustment phase of the washing run gave way to the production phase, which was continued as long as desired.[22]

[22] "Provision for continuous recirculation of the feed during the adjustment period and the use of a reserve portion of the feed for the test are of great advantage in conducting washing trials. Testing work performed in this manner constitutes the nearest approach to continuous operation as practiced in the plant that is attainable in the laboratory." Yancey, H. F., and Fraser, Thomas. Op. cit., p. 87.

YSES OF SAMPLES

9	10	11	12	13	14	15	16
St. Clair 6 C-2953	Saline 5 C-3024	Vermilion 7 C-3079	Sangamon 5 C-3132	Randolph 6 C-3204	Christian 6 C-3257	Williamson 6 C-3319	Knox 1 C-3463
9.9	6.1	11.0	13.5	9.8	11.6	8.2	12.7
19.3	9.8	17.4	13.9	15.9	12.1	7.9	11.5
34.4	33.2	35.0	32.8	33.0	34.9	31.4	35.8
36.4	50.9	36.6	39.8	41.3	41.4	52.5	40.0
21.5	10.4	19.5	16.1	17.6	13.7	8.6	13.2
38.1	35.4	39.3	38.0	36.6	39.5	34.3	41.0
40.4	54.2	41.2	45.9	45.8	46.8	57.1	45.8
48.6	39.5	48.9	45.2	44.4	45.7	37.5	47.3
51.4	60.5	51.1	54.8	55.6	54.3	62.5	52.7
5.16	5.31	5.44	5.45	5.25	5.45	5.34	5.90
54.75	68.78	57.05	57.58	57.65	58.60	68.90	59.18
0.99	1.91	1.06	1.07	1.14	0.98	1.62	0.99
14.77	11.77	15.18	17.65	16.42	19.04	15.36	17.86
4.97	2.47	3.88	4.37	3.69	3.77	.91	4.55
19.36	9.76	17.39	13.88	15.85	12.16	7.87	11 52
4.51	4.93	4.76	4.57	4.61	4.70	4.81	5.14
60.79	73.26	64.11	66.57	63.90	66.32	75.08	67.76
1.10	2.03	1.19	1.24	1.26	1.11	1.76	1.14
6.59	6.75	6.04	7.51	8.58	9.85	8.79	7.56
5.52	2.63	4.36	5.06	4.08	4.26	.99	5.21
21.49	10.40	19.54	16.05	17.57	13.76	8.57	13.19
5.74	5.50	5.91	5.44	5.60	5.45	5.27	5.92
77.41	81.79	79.66	79.30	77.53	76.88	82.14	78.06
1.41	2.27	1.48	1.48	1.53	1.29	1.93	1.31
8.41	7.51	7.53	7.76	10.38	11.44	9.57	8.71
7.03	2.93	5.42	6.02	4.96	4.94	1.09	6.00
9989	12333	10299	10354	10506	10720	12202	10919
11090	13136	11574	11970	11644	12132	13296	12501
14123	14666	14382	14260	14128	14064	14546	14403
1883	1920	1892	1873	1880	2016	2321	2044
1948	1940	1985	1935	1995	2054	2530	2099
2231	2287	2338	2327	2213	2359	2653	2289

At the end of the run all elements of the system were stopped simultaneously, to permit proper analysis of the separation taking place. Yield of cleaned coal was considered to be the ratio of cleaned coal produced to total coal separated (cleaned coal plus reject). However, it must be noted that total coal separated was always less than total coal fed, by the quantity of coal remaining throughout the system at the end of the run. This material, normally 150 to 250 lbs., accumulated on the table and in the recirculating system during the first period of the test, when washing equilibrium was being reached from an empty start. It was usually somewhat higher in ash content than the raw coal (tables 5, 6, and 7), primarily because of the high ash content of the bed of material on the deck of the table; and it was substantially con-

stant in nature and amount during any washing run. Because it was usually higher in ash content than the raw coal from which it was derived, the remainder of the coal—that is, the coal actually separated—was usually lower in ash content than the raw coal. This effect, causing the reported results in the laboratory to be slightly more favorable than would actually be the case if the washing operation were continued indefinitely, is reduced as the amount of coal fed is increased.

In the present investigation the quantities of coal handled were large enough to make the effect of the coal left in the system quite small. Table 3 gives the ash content of the material separated, computed by allowing for the higher ash content of the residual coal in the system. In only one case does this differ from the ash content of the raw coal sample by more than 0.9 percentage figure. It was concluded that the effect of coal left in the system on the results reported is not disturbingly large.

TABLE 3.—COMPARISON OF ASH CONTENT OF MATERIAL SEPARATED WITH THAT OF RAW COAL[a]

Run	Ash content, percent		Difference, percentage figures
	Material separated[b]	Raw coal	
Normal tabling			
51.....	18.5	19.0	+ 0.5
52.....	18.5	19.0	+ 0.5
61.....	17.9	18.1	+ 0.2
62.....	17.5	18.1	+ 0.6
71.....	14.2	14.3	+ 0.1
72.....	14.2	14.3	+ 0.1
81.....	22.5	23.3	+ 0.8
82.....	23.0	23.3	+ 0.3
91.....	19.9	21.5	+ 1.6
101.....	10.1	10.4	+ 0.3
102.....	10.1	10.4	+ 0.3
111.....	19.5	19.5	0.0
112.....	19.6	19.5	− 0.1
121.....	15.7	16.1	+ 0.4
122.....	15.6	16.1	+ 0.5
131.....	17.2	17.6	+ 0.4
141.....	13.6	13.7	+ 0.1
151.....	8.4	8.6	+ 0.2
152.....	8.5	8.6	+ 0.1
161.....	13.2	13.2	0.0
Retabling of previously tabled coal			
103.....	7.8	7.8	0.0
113.....	10.6	10.6	0.0
123.....	11.1	11.2	+ 0.1
133.....	12.6	12.6	0.0
143.....	9.2	9.2	0.0
Tabling of reject			
154.....	16.6	16.3	− 0.3
164.....	34.4	33.5	− 0.9

[a] Data on dry basis.
[b] Computed from ash content of raw coal by allowing for ash content of residual material in washing system.

The possibility of maintaining one or more elements of the washing operation constant was considered. Possible choices included: (1) Percentage of washed coal recovered from raw coal; (2) percentage of reduction of mineral matter or ash; (3) percentage of mineral matter or ash remaining in the washed coal; or (4) fixed adjustment of one or more of the operating variables of the table. None of these, however, seemed of general applicability to all Illinois coals, varying as they do in rank and in quantity and type of mineral matter. It was decided that each tabling run should be carried out with no restrictions on the operating adjustments, which were freely varied as needed to achieve visually good separation.

It is probable that the values obtained in the tablings were conservative compared to those obtainable in a properly controlled full-scale installation, using a larger table, unlimited raw coal of approximately constant physical characteristics, and unlimited time. Under such conditions, the cleaned coal product can be brought to an economically optimum quality and quantity by small refinements in operation, each readjustment being checked by sampling and chemical tests rather than by eye. Furthermore, the particular coal used may exhibit a pattern of separation which a change of riffling would assist. A larger table in itself aids separation by providing approximately twice the length of path for each particle before it finally reaches a discharge edge. From a quantitative standpoint, therefore, it may be assumed that any degree of improvement obtained in the present investigation could be at least equalled and probably excelled in a commercial plant.[23]

[23] "All test data (of this type) are influenced to a certain extent by such maladjustments as are difficult to eliminate in trial runs with washing machines, because often the operator has no means of knowing what kind of a separation is being made until the test is completed and the analyses are made." Yancey, H. F., and Fraser, Thomas. Op. cit., p. 87.

WASHABILITY ANALYSIS

Introduction

Washability analyses are regarded as indicative of best possible separation, and all proposed means of evaluating the efficiency of any observed separation are based on the theoretically possible separation revealed by a study of the washability data.

An analysis of washability characteristics is based upon the fortunate fact that all ash-forming materials found associated with coal are of higher specific gravity than the coal. Within the range of ash contents characteristic of coal the ash content is lineally related, for all practical purposes, to specific gravity.[24] It follows that any scheme which produces a perfect separation on the basis of specific gravity produces the best possible separation of low-ash from high-ash particles—that is, the greatest possible yield of coal of a given ash content, or the least possible average ash content for a given yield. A series of baths of heavy liquids, varying step-wise in specific gravity, is convenient for this purpose. The coal to be tested and analyzed is immersed in each of the baths in succession, care being taken to allow each particle an unrestricted opportunity to float or to sink. Fractions produced in this way may be analyzed for moisture, ash, and any other desired characteristics; and by proper use of the data it is possible to determine the amenability of the coal to separation. An analysis of this type is almost invariably made before a coal washing plant is designed.

Limitations and requirements of specific gravity tests

For the results to be of value, it is evident that the apparent specific gravity of the particles tested should not vary appreciably during the course of the test. However, it was found that the apparent specific gravity of particles of partly dried, normally high-moisture coal increased markedly when immersed in test liquids, rapidly at first and slowly reaching equilibrium. Such particles appeared to behave like porous solids, partly water filled. It is apparent that a porous solid, partly water filled, would temporarily exhibit a low specific gravity owing to the air in its interstices; then, as the test liquid replaced the air, bubbles would be emitted, with a progressive increase in apparent specific gravity. Replacement of air by test liquid would be rapid at first, but in later stages it would proceed more slowly. At the endpoint of complete replacement of air by test liquid, the apparent specific gravity of the solid would be at a maximum. Using a test liquid immiscible with water, the final stable apparent specific gravity would be higher as the water content of the porous solid is reduced.

Because the samples reserved for washability analyses, particularly those from high bed-moisture coals, exhibited precisely these characteristics, it was assumed that they were essentially porous solids, partly water-filled to a random extent depending upon how much they had dried. The magnitude of the apparent changes in specific gravity justify a search for means of attaining or at least approaching stability in this quality. In addition to being stable, the apparent specific gravity of an individual coal particle should be reproducible and reasonably similar to the specific gravity the coal particle might have had in a washing unit had it been subjected to normal handling and conveying from the coal face to the washer.

It is commonly agreed that coal in the bed is water-saturated,[25] and inasmuch as coal reaches the washing unit after only brief contact with dry air, it is probable that the majority of particles are nearly saturated at the time of washing. Accordingly, the specific gravity of water-saturated coal is to be desired in washability analyses.

Methods of treatment

In analyzing the ways of conducting a washability analysis, attention is confined to

[24] McCabe, Louis C., and Boley, Charles C. Physical properties of coals. Chap. 7 of Chemistry of coal utilization (H. H. Lowry, Ed.), Wiley (New York), 1868 pp., 1945; p. 313.

[25] Nebel, Merle L. Specific gravity studies of Illinois coal. Univ. Ill. Eng. Exp. Sta. Bull. 89, 49 pp., 1916.
Stansfield, Edgar, and Gilbart, K. C. Moisture determination for coal classification (authors' discussion). Trans. AIME Vol. 101, pp. 125–47, 1932; p. 147.
Bird, B. M. Discussion on paper by T. W. Guy, Determining surface moisture in coal. AIME Vol. 130, pp. 229–49, 1938; p. 249.

mixtures of carbon tetrachloride (CCl_4), benzene (C_6H_6), and bromoform ($CHBr_3$) only, and especially the first two named. These organic liquids, immiscible with water, are technically much superior to water solutions of zinc chloride or calcium chloride because they have low viscosity, high volatility, and are non-corrosive, although their cost is higher. It is assumed that the coal pores are small enough and the surface tension of the water within them is high enough so that the organic test liquids will not replace the water but will surround and hold it in the coal. That they do this is demonstrated by the fact that even surface water on coal particles immersed in mixtures of benzene and carbon tetrachloride clings to the coal surfaces.

There are three general methods of handling a coal sample in connection with a washability analysis:

(1) No preadjustment of moisture content. The particles are taken as they happen to be in the laboratory and are allowed to soak in test liquid of low specific gravity for any length of time necessary for all air to be displaced.

Advantages: The method is simple. Specific gravity of any particle is stable after all air has been displaced, providing no change is made in the specific gravity of the test liquid.

Disadvantages: Specific gravity is always higher than the desired water-saturated specific gravity. The differential increases with increased drying, as a consequence of reduction in material of 1.0 specific gravity (water).

Specific gravity becomes temporarily unstable with each change in specific gravity of test liquid, until new test liquid and old test liquid remaining in pores of coal particles reach equilibrium.

Specific gravity is not reproducible, varying with extent of prior air drying, which in general is not controllable or known.

(2) Preadjustment of moisture content to minimum possible, before immersion in test liquid of low specific gravity, followed by a period of soaking to displace all air.

Advantages: Specific gravity of any particle is stable after all air has been displaced, providing no change is made in the specific gravity of the test liquid.

Specific gravity is reproducible.

Disadvantages: The method is more complex than (1).

Specific gravity is always higher than the desired water-saturated specific gravity.

Specific gravity becomes temporarily unstable with each change in specific gravity of test liquid,

(3) Preadjustment of moisture content to maximum possible (that is, water saturation), before immersion in test liquid.

Advantages: Specific gravity is stable.

Specific gravity is reproducible.

Specific gravity is theoretically identical with, and at least very close to, the desired water-saturated specific gravity.

Specific gravity does not become temporarily unstable with each change in specific gravity of test liquid.

No soaking period is required in test liquid, to displace air.

Disadvantages: The method is complex.

Surface moisture on the coal particles after saturating with water must be removed.

Soaking in water causes some disintegration of clay, shale, and coal.

From this cataloging of the respective advantages and disadvantages of the three general methods, it appears that the last-named, involving saturating coal particles with water, is most likely to give both a stable and a reproducible specific gravity value corresponding reasonably well to that which a particle would have exhibited in a washing operation under normal conditions.

Surface moisture under the water-saturation method is a disadvantage. Unless it is removed the resulting errors are intolerable when test liquids immiscible with water are used, because of the agglomeration of particles and also because of lowered apparent specific gravity due to excess water.

When water-miscible solutions of inorganic salts, such as zinc chloride, are used, surface moisture is immediately taken into solution and causes no apparent difficulty. However, the apparent specific gravity of a particle steadily increases as the salt solution diffuses into it, and the final stable specific gravity is always higher than the desired water-saturated specific gravity.

The use of water-saturated coal in water-immiscible test liquid thus seemed indicated, providing that a satisfactory method could be found for removing surface moisture.

Removal of surface moisture

The ideal condition of complete water saturation of a mass of particles, with simultaneous lack of surface moisture, is probably unachievable in a strict sense, although it appears to be theoretically desirable. Three methods were considered for the removal

of surface moisture: (1) The use of a diluent (alcohol) miscible both in water and in carbon tetrachloride-benzene mixtures; (2) the use of a wetting agent for reducing surface tension of water, so that the test liquid could preferentially wet the coal surface and displace the water film; and (3) the use of a mild current of air to dry the particles to visual surface dryness, followed by their immediate immersion in test liquid.

Methods (1) and (2) as listed above have been investigated rather thoroughly in a study closely related to the work herein reported,[26] and either can be used for the original purpose. The techniques involved are fairly complex when compared with usual float-and-sink methods but are entirely feasible and objective. However, both methods were found to alter the desired condition of water saturation of coal by permitting the interstitial water to assume the specific gravity of the test liquid or at least to tend in that direction. Neither alcohol, in taking water into solution, nor a wetting agent, in reducing the surface tension of water to permit wetting by the organic test liquid, can be expected to discriminate between surface and interstitial water. Thus, despite their apparent objectivity and success in removing surface moisture, it was concluded that methods (1) and (2) do not fulfill the desired conditions.

Method (3), although tedious and rather subjective, was felt to be superior and was used in obtaining all the washability data presented.

Procedure adopted

A representative portion of the head sample, approximately 1000 grams in weight, was prepared by riffling, and the material less than 20-mesh in size was removed by screening. Nominally, there should have been no material less than 20-mesh, inasmuch as all coal had previously been screened for minimum size at 8-mesh (p. 14). However, no continuous screening process is perfect, and further screening will always produce more undersize, undoubtedly the result in part of the breaking action of the screening process. The removal of the undersize, usually less than two percent (table 4), greatly simplified the later tasks of surface drying and of handling in organic solutions, without unduly affecting the float-and-sink results.[27]

Table 4.—Material in Head Samples not Included in Washability Analysis

Coal	Fines, percent[a]	Clay, percent[b]	Total fines and clay, percent	Ash content, percent
5	1.7	4.4	6.1	62.4
6	1.4	5.8	7.2	72.2
7	(c)	(c)	(c)	(c)
8	2.1	2.2	4.3	54.5
9	1.3	0.7	2.0	32.1
10	0.3	0.5	0.8	18.1
11	1.1	1.2	2.3	46.1
12	1.1	2.6	3.7	63.9
13	0.9	2.0	2.9	64.6
14	(c)	(c)	3.6	20.0
15	1.2	0.1	1.3	13.0

[a] Material through 20-mesh screen, before water soaking.
[b] Material through 50-mesh screen, after water soaking and during surface drying.
[c] Not obtained.

The "dedusted" sample was soaked in water for at least 24 hours. It was then removed, a few particles at a time, and spread out on a 50-mesh screen under close observation in a mild current of air. As the particles lost their surface film of water, they were selectively removed and immersed in a mixture of carbon tetrachloride and benzene, adjusted to about 1.25 specific gravity. Throughout succeeding operations they were not exposed to the atmosphere again until they were ready to be removed as a float product. In this respect the procedure differed from that commonly employed, wherein each sink fraction is removed by straining and filtering, and then reintroduced into the liquid of next heavier specific gravity.

In the procedure here adopted, after the initial immersion of particles in test liquid, the specific gravity of the bath was increased by the addition of appropriate amounts of denser liquids, with stirring, until a convenient amount of coal floated. This was

[26] Godoy-Peralta, Otto Israel. An investigation of soaking Illinois coal in water as a preliminary step in float and sink testing. Thesis submitted in partial fulfillment of requirements for degree of Bachelor of Science in Mining Engineering, Univ. of Illinois, 1945, 44 pp.

[27] Yancey, H. F., and Fraser, Thomas. Op. cit., p. 123.

skimmed off, with precautions to prevent entrapment in the float of material denser than the test liquid, and the specific gravity of the test liquid was determined by a certified hydrometer, usually by withdrawing the necessary amount of liquid in a pipette and testing it in a separate cylinder. This cycle was repeated, each time floating as much or as little coal as convenient, until the entire range of specific gravities appropriate for the coal was covered.

In this procedure the common specific gravity steps of 1.30, 1.40, 1.50, and 1.70 were not directly used. However, information on the relative quantities floating and sinking at these or any other specific gravities is readily obtained from smooth curves drawn through plots of the observed specific gravities and the corresponding yield. Furthermore, the incremental changes of specific gravity used were those best fitted in each case to the particular coal, which may have been of such a character that rather large or rather small increments were appropriate in certain portions of the range.

The major advantage of the procedure was that it prevented partial drying of particles between baths, this drying being of unknown and variable extent.

Soaking in water to establish a water-saturated condition disintegrates a certain amount of clay and shale, which sifts through the screen used during the surface drying. This material was added to the minus 20-mesh dust obtained in the dedusting operation, and the whole was analyzed for moisture and ash. The amount of material from these two sources, not subjected to float-and-sink, is shown in table 4.

The procedure is tedious and, in some respects, subjective; but it is felt that for coals of high bed-moisture it is superior to the usual scheme of immersion in test liquid with no moisture control because it produces a much closer approach to apparent specific gravities which are stable, reproducible, and similar to those which the same particles would have had in the normal sequence of mining, conveying, and washing. The procedure is not intended for coals of low bed-moisture, nor is it suggested for control work in preparation plants.

Results

Changes of quality produced by tabling

Table 5 presents a summary of data relating to yield and quality in the head samples and the products resulting from 20 washing tests conducted on 12 Illinois raw coals with the equipment described. Table 7 presents similar data on rewashing tests run on five of the washed coals, and table 6 presents data on rewashing two lots of reject. All data are reported on the dry basis.

In these tables the product called "system" is the material that was in various parts of the washing system and so not assignable to either the washed coal or the reject when the test was finished—as, for example, the material on the deck of the table. The product called "loss" is the difference between the dry weight of coal fed to the table and the sum of the dry weights of the products. It is usually less than one percent of the feed weight; or in other words, the material balance on weight of coal is usually 99 percent or better (table 8). The material balance on weight of ash is usually poorer, ranging from 82.5 to 109.4 percent (table 8). The only apparent explanation of such variations is one of accumulated permissible errors in the sampling and in the ash determinations. The American Society for Testing Materials Tentative Standard D492-40T, "Tentative method of sampling coals classed according to ash content," establishes a sampling procedure such that a single test result is expected, with a probability of 0.95, to be within plus or minus 10 percent of the true ash content. It can be shown that all material balances in table 8 on weight of ash may be adjusted to 100 percent, if desired, by taking advantage of a tolerance of plus or minus 10 percent of the several reported ash contents.

Table 8 also compares the extent of quality improvement with respect to ash, sulfur, mineral matter, and heating value secured by tabling, and indicates the percentage of pure coal (unit coal)[28] and of

[28] Parr, S. W., and Wheeler, W. F. Unit coal and the composition of coal ash. Univ. Ill. Eng. Exp. Sta. Bull. 37, 67 pp., 1909.

TABLE 5.—TABLING OF 12 RAW COALS[a]

Products	Weight		Ash, percent	Sulfur, percent	Mineral matter,[b] percent	Heating value, B.t.u./lb.	Yield,[c] percent
	lb.	percent					
Run 51—Macoupin County—Coal seam No. 6							
Raw	1664	—	19.0	5.47	23.5	11387	—
Washed	1371	82.4	9.8	3.97	12.8	12594	90.9
Reject	137	8.2	64.8	—	—	—	9.1
System[d]	155	9.3	24.3	—	—	—	—
Loss	1	0.1	—	—	—	—	—
Run 52—Macoupin County—Coal seam No. 6							
Raw	1975	—	19.0	5.47	23.5	11387	—
Washed	1598	80.9	9.4	3.92	12.3	12618	88.8
Reject	202	10.2	62.2	—	—	—	11.2
System[d]	174	8.8	23.8	—	—	—	—
Loss	1	0.1	—	—	—	—	—
Run 61—Peoria County—Coal seam No. 5							
Raw	1896	—	18.1	4.13	21.8	11776	—
Washed	1533	80.9	10.6	3.37	13.3	12763	92.6
Reject	123	6.5	72.7	—	—	—	7.4
System[d]	249	13.1	19.6	—	—	—	—
Loss	−9	−0.5	—	—	—	—	—
Run 62—Peoria County—Coal seam No. 5							
Raw	1840	—	18.1	4.13	21.8	11776	—
Washed	1594	86.6	11.1	3.44	13.9	12552	95.2
Reject	80	4.4	77.9	—	—	—	4.8
System[d]	175	9.5	23.8	—	—	—	—
Loss	−9	−0.5	—	—	—	—	—
Run 71—Gallatin County—Coal seam No. 5							
Raw	1828	—	14.3	5.35	18.4	12649	—
Washed	1449	79.3	10.7	3.54	13.5	13408	92.1
Reject	124	6.8	45.4	—	—	—	7.9
System[d]	229	12.5	14.8	—	—	—	—
Loss	26	1.4	—	—	—	—	—
Run 72—Gallatin County—Coal seam No. 5							
Raw	2080	—	14.3	5.35	18.4	12649	—
Washed	1332	64.0	10.0	3.42	12.7	13505	75.6
Reject	429	20.6	24.6	—	—	—	24.4
System[d]	293	14.1	14.9	—	—	—	—
Loss	26	1.3	—	—	—	—	—
Run 81—Wabash County—Friendsville coal							
Raw	1732	—	23.3	3.09	26.9	10794	—
Washed	1356	78.3	15.0	2.36	17.5	11953	89.9
Reject	152	8.8	57.9	—	—	—	10.1
System[d]	197	11.4	29.9	—	—	—	—
Loss	27	1.5	—	—	—	—	—
Run 82—Wabash County—Friendsville coal							
Raw	1907	—	23.3	3.09	26.9	10794	—
Washed	1364	71.5	12.9	2.33	15.2	12290	79.4
Reject	353	18.5	43.5	—	—	—	20.6
System[d]	163	8.6	26.5	—	—	—	—
Loss	27	1.4	—	—	—	—	—

TABLE 5.—(Continued)

Products	Weight		Ash, percent	Sulfur, percent	Mineral matter,[b] percent	Heating value, B.t.u./lb.	Yield,[c] percent
	lb.	percent					
Run 91—St. Clair County—Coal seam No. 6							
Raw	1857	—	21.5	5.52	26.3	11090	—
Washed	1465	78.9	13.1	3.62	16.1	12362	91.1
Reject	143	7.7	62.3	—	—	—	8.9
System[d]	187	10.1	35.9	—	—	—	—
Loss	62	3.3	—	—	—	—	—
Run 101—Saline County—Coal seam No. 5							
Raw	2311	—	10.4	2.63	12.7	13136	—
Washed	1831	79.2	7.6	2.06	9.3	13573	87.4
Reject	265	11.5	36.8	—	—	—	12.6
System[d]	199	8.6	13.2	—	—	—	—
Loss	16	0.7	—	—	—	—	—
Run 102—Saline County—Coal seam No. 5							
Raw	3667	—	10.4	2.63	12.7	13136	—
Washed	3034	82.7	7.8	2.10	9.6	13532	89.5
Reject	354	9.7	39.3	—	—	—	10.5
System[d]	254	6.9	14.4	—	—	—	—
Loss	25	0.7	—	—	—	—	—
Run 111—Vermilion County—Coal seam No. 7							
Raw	1868	—	19.5	4.36	23.5	11574	—
Washed	1466	78.5	11.4	3.53	14.3	12925	86.0
Reject	238	12.7	61.8	—	—	—	14.0
System[d]	164	8.8	19.3	—	—	—	—
Loss	—	—	—	—	—	—	—
Run 112—Vermilion County—Coal seam No. 7							
Raw	3421	—	19.5	4.36	23.5	11574	—
Washed	2680	78.3	10.6	3.40	13.3	13064	82.3
Reject	578	16.9	57.6	—	—	—	17.7
System[d]	165	4.8	17.9	—	—	—	—
Loss	−2	—	—	—	—	—	—
Run 121—Sangamon County—Coal seam No. 5							
Raw	2797	—	16.1	5.06	20.2	11970	—
Washed	2183	78.0	11.1	4.49	14.5	12659	86.8
Reject	331	11.8	37.9	—	—	—	13.2
System[d]	220	7.9	20.5	—	—	—	—
Loss	63	2.3	—	—	—	—	—
Run 122—Sangamon County—Coal seam No. 5							
Raw	1825	—	16.1	5.06	20.2	11970	—
Washed	1431	78.4	11.2	4.49	14.6	12718	91.1
Reject	140	7.7	49.4	—	—	—	8.9
System[d]	215	11.8	19.7	—	—	—	—
Loss	39	2.1	—	—	—	—	—
Run 131—Randolph County—Coal seam No. 6							
Raw	4697	—	17.6	4.08	21.3	11644	—
Washed	3889	82.8	12.6	3.20	15.4	12360	88.3
Reject	515	11.0	54.3	—	—	—	11.7
System[d]	270	5.7	23.7	—	—	—	—
Loss	23	0.5	—	—	—	—	—

TABLE 5.—(Concluded)

Products	Weight		Ash, percent	Sulfur, percent	Mineral matter,[b] percent	Heating value, B.t.u./lb.	Yield,[c] percent
	lb.	percent					
Run 141—Christian County—Coal seam No. 6							
Raw	5934	—	13.7	4.26	17.1	12132	—
Washed	4768	80.4	9.2	3.90	12.1	12816	86.8
Reject	725	12.2	40.5	11.92	50.3	—	13.2
System[d]	356	6.0	15.3	—	—	—	—
Loss	85	1.4	—	—	—	—	—
Run 151—Williamson County—Coal seam No. 6							
Raw	3324	—	8.6	0.99	9.8	13296	—
Washed	2639	79.4	7.4	0.92	8.5	13407	85.7
Reject	442	13.3	17.9	1.68	20.3	—	14.3
System[d]	211	6.3	11.0	—	—	—	—
Loss	32	1.0	—	—	—	—	—
Run 152—Williamson County—Coal seam No. 6							
Raw	3147	—	8.6	0.99	9.8	13296	—
Washed	2374	75.5	7.0	0.91	8.1	13543	81.1
Reject	555	17.6	15.1	1.45	17.1	—	18.9
System[d]	189	6.0	10.0	—	—	—	—
Loss	29	0.9	—	—	—	—	—
Run 161—Knox County—Coal seam No. 1							
Raw	4588	—	13.2	5.21	17.1	12501	—
Washed	3559	77.6	8.9	3.75	11.7	13128	81.8
Reject	794	17.3	33.5	10.9	42.2	—	18.2
System[d]	222	4.8	14.1	—	—	—	—
Loss	13	0.3	—	—	—	—	—

[a] All data are reported on dry basis.
[b] Defined as (1.08 × ash plus 0.55 × sulfur).
[c] Referred to material separated (sum of washed coal and reject).
[d] Material in washing system at termination of operation.

TABLE 6.—RETABLING OF TWO REJECTS[a]

Products	Weight		Ash, percent	Sulfur, percent	Mineral matter,[b] percent	Heating value, B.t.u./lb.	Yield,[c] percent
	lb.	percent					
Run 154—Williamson County—Coal seam No. 6 (reject material, 42 percent from Run 151 and 58 percent from Run 152)							
Head	879	—	16.3	1.55	18.5	—	—
Washed	535	60.9	9.0	0.98	10.3	—	78.6
Reject	146	16.6	45.7	3.63	51.4	—	21.4
System[d]	197	22.4	15.4	—	—	—	—
Loss	1	0.1	—	—	—	—	—
Run 164—Knox County—Coal seam No. 1 (reject material from Run 161)							
Head	627	—	33.5	10.9	42.2	—	—
Washed	274	43.7	18.3	5.95	23.0	—	56.1
Reject	214	34.1	52.8	—	—	—	43.9
System[d]	140	22.3	30.5	—	—	—	—
Loss	−1	−0.1	—	—	—	—	—

[a] All data are reported on dry basis.
[b] Defined as (1.08 × ash + 0.55 × sulfur).
[c] Referred to material separated (sum of washed coal and reject).
[d] Material in washing system at termination of operation.

TABLE 7.—RETABLING OF FIVE WASHED COALS[a]

Products	Weight		Ash, percent	Sulfur, percent	Mineral matter,[b] percent	Heating value, B.t.u./lb.	Yield,[c] percent
	lb.	percent					
Run 103—Saline County—Coal seam No. 5 (washed coal from Run 102)							
Head	3239	—	7.8	2.10	9.6	13536	—
Washed	1385	42.8	6.3	1.84	7.8	13725	45.4
Reject	1669	51.5	8.5	—	—	—	54.6
System[d]	173	5.3	8.3	—	—	—	—
Loss	12	0.4	—	—	—	—	—
Run 113—Vermilion County—Coal seam No. 7 (washed coal from Run 112)							
Head	2552	—	10.6	3.40	13.3	13064	—
Washed	1558	61.1	9.1	3.25	11.6	13290	69.1
Reject	697	27.3	14.0	—	—	—	30.9
System[d]	158	6.2	11.0	—	—	—	—
Loss	139	5.4	—	—	—	—	—
Run 123—Sangamon County—Coal seam No. 5 (washed coal, 36 percent from Run 121 and 64 percent from Run 122)							
Head	1923	—	11.2	4.49	14.6	12697	—
Washed	1193	62.0	10.1	4.16	13.2	12851	71.2
Reject	482	25.1	14.7	—	—	—	28.8
System[d]	209	10.9	11.9	—	—	—	—
Loss	39	2.0	—	—	—	—	—
Run 133—Randolph County—Coal seam No. 6 (washed coal from 131)							
Head	2257	—	12.6	3.20	15.4	12360	—
Washed	1401	62.1	10.1	2.99	12.6	12753	67.9
Reject	661	29.3	16.9	3.61	20.2	—	32.1
System[d]	187	8.3	12.8	—	—	—	—
Loss	8	0.3	—	—	—	—	—
Run 143—Christian County—Coal seam No. 6 (washed coal from Run 141)							
Head	2033	—	9.2	3.90	12.1	12816	—
Washed	1295	63.7	8.2	3.68	10.9	12978	68.9
Reject	584	28.7	11.4	4.04	14.5	—	31.1
System[d]	139	6.9	9.1	—	—	—	—
Loss	15	0.7	—	—	—	—	—

[a] All data are reported on dry basis.
[b] Defined as (1.08 × ash plus 0.55 × sulfur).
[c] Referred to material separated (sum of washed coal and reject).
[d] Material in washing system at termination of operation.

heat units recovered by the washed coal from the feed coal. It will be noted that in several cases one or both of the latter percentages exceed 100, which is theoretically impossible. The vagaries of sampling must also be held accountable for this, but in addition, there is another explanatory circumstance; the percentage of yield of washed coal, as used for these computations was calculated on the basis of *coal separated.* As has been pointed out, the *coal separated* is nearly always slightly cleaner than *raw coal,* because of the high-ash material remaining on the table and in other parts of the washing system at the end of each run. Hence, the computed recoveries of pure coal and of heat units are slightly larger than they would have been if an indefinitely large quantity of coal had been separated, as in plant operation.

In run 72 the same raw coal was used as in run 71, but a greater percentage of material was rejected by appropriate adjustments of the operating variables, in an endeavor to produce a washed coal markedly superior to that produced in run 71, which was regarded as more nearly "normal."

TABLE 8.—MATERIAL BALANCES, QUALITY IMPROVEMENT, AND RECOVERY FOR WASHING RUNS ON RAW COALS[a]

Run	Material balances, output to input		Quality improvement, change as a percentage of original value				Recovery in washed coal	
	Feed weight, percent	Ash weight, percent	Ash	Sulfur	Mineral matter	Heating value	Unit coal,[b] percent	Heat units, percent
51	99.9	82.5	48.8	27.4	45.7	10.6	103.6	100.5
52	99.9	84.5	50.5	28.3	47.7	10.8	101.8	98.4
61	100.5	87.6	41.9	16.7	39.0	8.4	102.7	100.4
62	100.5	84.3	38.7	18.4	36.4	6.6	104.8	101.5
71	98.6	93.8	25.2	33.8	26.6	6.0	97.6	97.6
72	98.7	94.9	30.5	36.1	31.0	6.8	80.9	80.7
81	98.5	86.8	35.6	23.6	34.8	10.7	101.5	99.6
82	98.6	83.9	44.7	24.6	43.4	13.9	92.1	90.4
91	96.7	87.2	39.1	34.4	38.5	11.5	103.7	101.5
101	99.3	109.4	26.9	21.7	26.3	3.3	90.8	90.3
102	99.3	108.1	25.0	20.2	24.4	3.0	92.7	92.2
111	100.0	94.9	41.5	19.0	39.3	11.7	96.3	96.0
112	100.0	96.9	45.6	22.0	43.4	12.9	93.3	92.9
121	97.7	91.7	31.1	11.3	28.3	5.8	93.0	91.8
122	97.9	92.5	30.4	11.3	27.7	6.2	97.5	96.8
131	99.5	100.8	28.4	21.6	27.7	6.1	94.9	93.7
141	98.6	96.8	32.8	8.5	29.5	5.6	92.0	91.7
151	99.0	104.1	14.0	7.1	13.5	0.8	86.9	86.4
152	99.1	99.4	18.6	8.1	17.3	1.9	82.6	82.6
161	99.7	101.4	32.6	28 0	31.8	5.0	87.1	85.9

[a] All data on dry basis.
[b] Unit coal = whole coal less mineral matter = whole coal less (1.08 × ash plus 0.55 × sulfur).

TABLE 9.—TABLING WITH "NORMAL" AND HIGH PERCENTAGES OF REJECTS[a]

	Coal 7			Coal 8		
	Raw	First tabling (Run 71)	Second tabling (Run 72)	Raw	First tabling (Run 81)	Second tabling (Run 82)
Reject, percent		7.9	24.4		10.1	20.6
Weight yield, percent		92.1	75.6		89.9	79.4
Heat unit yield, percent		97.6	80.7		99.6	90.4
Analysis:						
Ash, percent	14.3	10.7	10.0	23.3	15.0	12.9
Sulfur, percent	5.35	3.54	3.42	3.09	2.36	2.33
Mineral matter, percent	18.4	13.5	12.7	26.9	17.5	15.2
Heating value, B.t.u./lb.	12649	13408	13505	10794	11953	12290
Quality improvement:[b]						
Ash, percent		25.2	30.5		35.6	44.7
Sulfur, percent		33.8	36.1		23.6	24.6
Mineral matter, percent		26.6	31.0		34.8	43.4
Heating value, percent		6.0	6.8		10.7	13.9

[a] All data on dry basis.
[b] Computed as a percentage of change in original value.

Run 82 compares similarly with run 81. Table 9 shows that washed coals of only slightly higher quality resulted, and attempts were then made to secure extremes of high quality by retabling some of the tabled products.

Table 10 gives material balances, quality improvement, and recovery values obtained by retabling five coals. The operating variables were adjusted to give as large a percentage of reject as possible, consistent with the need for about 1500 pounds of especially

TABLE 10.—MATERIAL BALANCES, QUALITY IMPROVEMENT, AND RECOVERY FOR RETABLINGS OF FIVE WASHED COALS[a]

Run	Material balances, output to input		Quality improvement, change as a percentage of value in feed to table				Recovery in washed coal	
	Feed weight, percent	Ash weight, percent	Ash	Sulfur	Mineral matter	Heating value	Unit coal,[b] percent	Heat units, percent
103	99.6	96.4	17.1	12.4	18.8	1.4	46.3	46.0
113	94.6	94.9	20.2	4.4	12.8	1.7	70.5	70.3
123	98.0	100.4	9.0	7.3	9.6	1.2	72.4	71.9
133	99.7	97.5	19.8	6.6	18.2	3.2	70.1	70.1
143	99.3	98.7	10.9	5.6	9.9	1.3	69.8	69.8

[a] All data on dry basis.
[b] Unit coal = whole coal less mineral matter = whole coal less (1.08 × ash plus 0.55 × sulfur).

well cleaned coal for testing in the stoker-boiler unit. In other words, the best possible 1500 pounds was desired, and the table was used to recover this amount, regardless of variations in the total amount of coal available at this point in the program.

Table 10 shows that the further increases in heating value effected by retabling washed coal were small (ranging from 1.2 to 3.2 percent), at the expense of very low recoveries in total heat units (ranging from 46.0 to 71.9 percent).

Because the attempts to secure extremes of high quality both by tabling raw coal with a high percentage of reject (table 9) and by retabling previously washed coals (table 10) did not give as large an improvement in quality as had been desired, it was decided to use large-scale float-and-sink methods, theoretically superior but much more laborious. Work of this type was done, but the results do not appear to be pertinent to the present report and are not given herein.

SETTINGS OF OPERATING VARIABLES

The quality changes reported in the preceding section were obtained with table settings as given in table 11, in which the designations are self-explanatory with the possible exception of "composite slope." Composite slope represents an attempt to combine into a single figure the probable joint effect of transverse slope and longitudinal slope on the flow of material over the deck. It is the slope of the deck taken in a direction perpendicular to the path of a particle moving in a straight line from the feed corner to the corner diagonally opposite. Theoretically, tendency for material to move to the cleaned coal edge should increase with increasing composite slope, providing transverse and longitudinal slopes are within their normal ranges of variation.

WASHABILITY DATA

Each of the 12 coals washed (with one exception) was analyzed for washability characteristics by the procedure outlined. All specific gravity fractions so obtained were dried of test liquid and analyzed for moisture and ash. Results were computed to the moisture-free basis, and the data are presented in tables 12 to 22, and in figures 4 to 9. In these figures there is no significance in the pairing of data.

As a rule, the ash content of the head sample as analyzed does not check exactly with the ash content of the head sample as composited from the data of the specific gravity fractions, owing to fortuitous accumulations of sampling and analytical errors. An adjustment was made to bring the composited ash value of the head sample into agreement with the directly analyzed head sample value by an appropriate increase in the weight percentage of the heaviest sink fraction. This made allowance for the high-ash fine material which was not floated (table 4) and is in accordance with the methods used by Yancey and Fraser.[29]

[29] Yancey, H. F., and Fraser, Thomas. Op. cit., p. 88.

TABLE 11.—SETTINGS OF OPERATING VARIABLES

Run	Coal feed, lb./min.	Rate of introduction of wash water, gal. min.	Ratio of water to coal, by weight	Stroke, inches	Rate of reciprocation of deck, cycles/min.	Total deck movement, in./min.	Transverse slope, degrees	Longitudinal slope, degrees	Composite slope,[a] degrees
Tabling of 12 raw coals[c]									
51	28.2	41.4	12.24	1.25	224	560	5.35	2.77	4.10
52	25.3	42.6	14.05	1.25	224	560	2.67	2.77	2.72
61	61.4	([b])	([b])	0.69	282	388	4.50	1.98	3.28
62	59.7	46.9	6.72	0.69	282	388	2.75	2.23	2.50
71	39.7	38.9	8.17	0.88	268	469	4.58	1.97	3.32
72	39.7	40.5	8.48	0.88	290	508	2.08	1.17	1.63
81	29.5	29.5	8.33	0.88	272	476	5.42	1.23	3.38
82	33.7	33.7	8.33	0.88	272	476	2.40	0.92	1.60
91	38.1	37.4	8.18	0.88	272	476	4.17	1.07	2.67
101	53.4	30.4	4.74	1.00	272	544	3.70	0.98	2.38
102	38.6	30.4	6.56	1.00	272	544	3.70	0.98	2.38
111	35.4	32.7	7.70	1.00	272	544	2.75	1.03	1.92
112	21.9	33.8	12.87	1.00	272	544	2.75	1.03	1.92
121	45.2	32.5	6.00	1.00	280	560	2.75	0.83	1.82
122	35.8	33.9	7.90	1.00	280	560	2.67	0.83	1.77
131	37.4	35.1	7.83	1.00	282	564	3.02	0.75	1.92
141	28.0	25.9	7.72	1.06	278	591	2.83	0.78	1.83
151	31.1	21.7	5.82	1.06	278	591	2.13	0.93	1.55
152	35.6	21.8	5.10	1.06	278	591	2.13	0.93	1.55
161	24.4	26.4	9.01	1.00	278	556	2.00	0.78	1.40
Retabling of five washed coals[d]									
103	43.7	27.4	5.23	1.00	272	544	1.83	0.37	1.12
113	31.7	27.3	7.18	1.00	272	544	2.08	0.55	1.33
123	35.6	30.5	7.15	1.00	280	560	2.00	0.62	1.33
133	30.6	25.2	6.86	1.00	282	564	2.92	0.67	1.83
143	27.5	25.2	7.64	1.06	278	591	1.58	0.73	1.10
Retabling of two rejects[e]									
154	26.8	24.3	7.56	1.00	278	556	2.70	0.90	1.82
164	20.8	24.0	9.62	1.00	278	556	2.83	0.87	1.88

[a] Slope of table in direction perpendicular to path of particle travelling directly from feed corner to diagonally opposite corner. See page 28.
[b] Not obtained.
[c] See table 5 for coals used and results of runs.
[d] See table 7 for coals used and results of runs.
[e] See table 6 for coals used and results of runs.

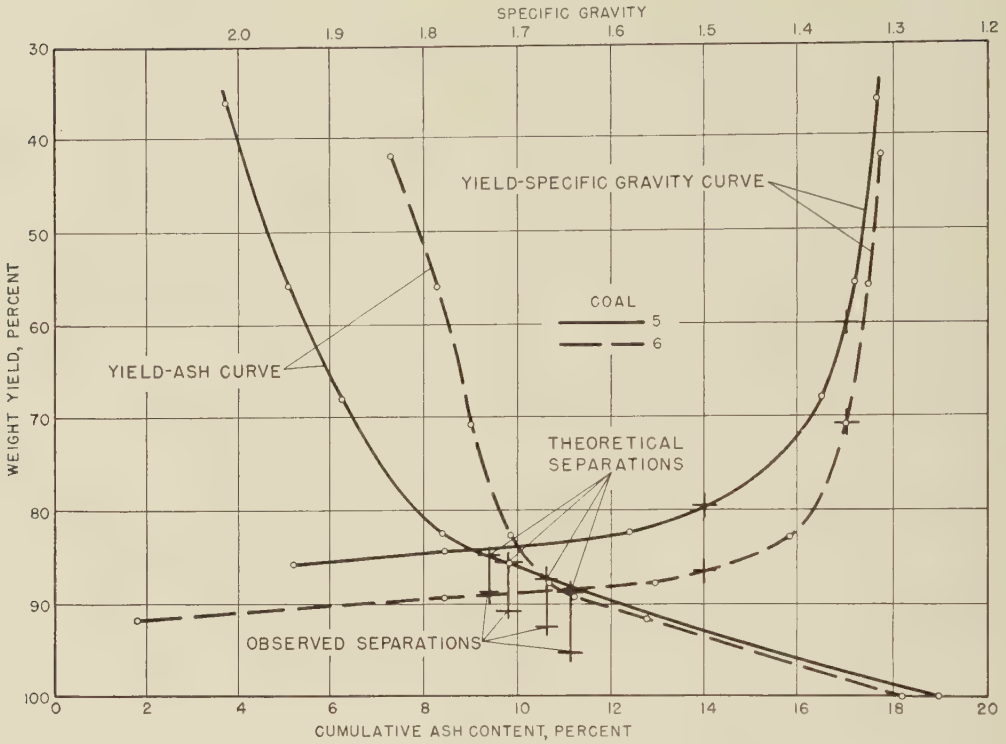

FIG. 4.—Yield-ash and yield-specific gravity curves for coals 5 and 6.

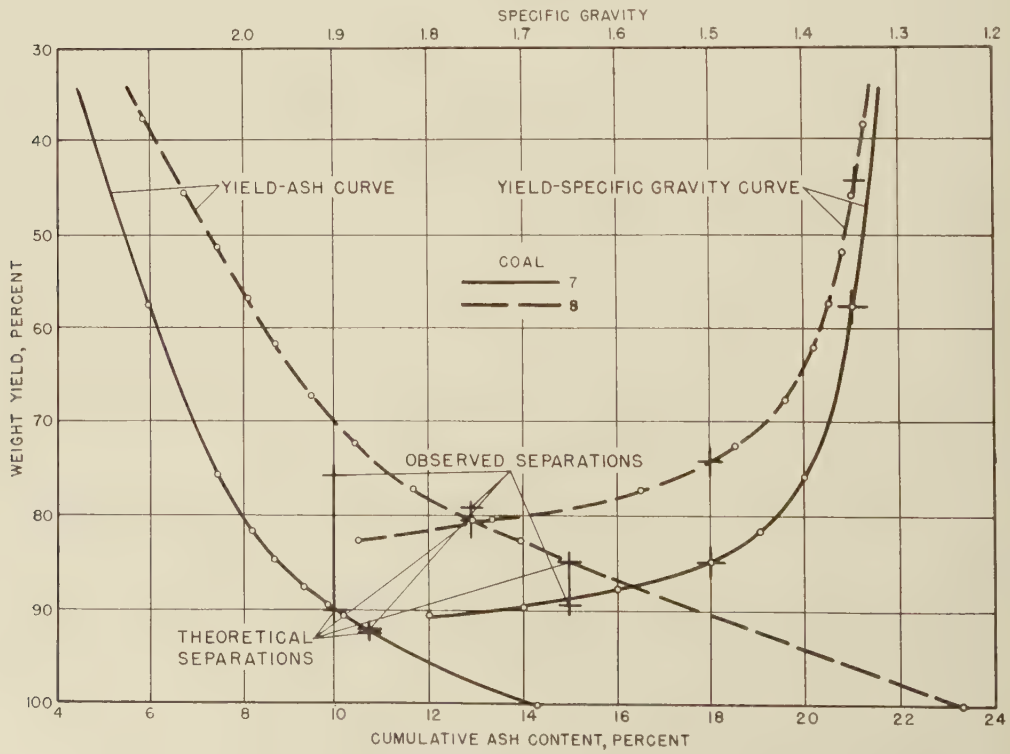

FIG. 5.—Yield-ash and yield-specific gravity curves for coals 7 and 8.

TABLE 12.—WASHABILITY DATA[a] FOR COAL 5

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.288 Float	7.69	2.5	7.69	7.34	2.50
1.288 S—1.300 F	15.90	3.4	23.59	22.52	3.11
1.300 S—1.316 F	14.11	4.9	37.70	36.00	3.78
1.316 S—1.340 F	20.48	7.5	58.18	55.55	5.09
1.340 S—1.374 F	12.90	11.5	71.08	67.87	6.25
1.374 S—1.580 F	15.11	18.5	86.19	82.30	8.40
1.580 S—1.778 F	1.99	41.0	88.18	84.20	9.13
1.778 S—1.940 F	1.48	53.6	89.66	85.61	9.87
1.940 Sink	10.34	73.3	100.00	100.00	19.00

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 19.0 percent (laboratory no. C-2697, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.940 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 13.—WASHABILITY DATA[a] FOR COAL 6

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.280 Float	8.30	4.4	8.30	8.01	4.40
1.280 S—1.294 F	10.78	5.9	19.08	18.42	5.25
1.294 S—1.314 F	24.34	8.8	43.42	41.93	7.24
1.314 S—1.326 F	14.44	11.3	57.86	55.87	8.25
1.326 S—1.350 F	15.27	11.8	73.13	70.62	8.99
1.350 S—1.410 F	12.41	15.0	85.54	82.60	9.86
1.410 S—1.554 F	5.03	24.1	90.57	87.46	10.66
1.554 S—1.780 F	1.71	39.3	92.28	89.11	11.19
1.780 S—2.110 F	2.57	70.1	94.85	91.59	12.78
2.110 Sink	5.15	76.0	100.00	100.00	18.10

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 18.1 percent (laboratory no. C-2776, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (2.110 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 14.—WASHABILITY DATA[a] FOR COAL 7

Specific gravity fractions[c]	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.2875 Float	7.27	2.71	7.27	7.13	2.71
1.2875 S—1.30 F	9.24	3.69	16.51	16.19	3.26
1.30 S—1.35 F	42.37	7.01	58.88	57.75	5.96
1.35 S—1.40 F	18.24	12.39	77.12	75.64	7.48
1.40 S—1.45 F	6.20	17.15	83.32	81.73	8.20
1.45 S—1.50 F	3.19	21.49	86.51	84.86	8.69
1.50 S—1.60 F	3.07	26.78	89.58	87.87	9.31
1.60 S—1.70 F	1.82	34.42	91.40	89.65	9.81
1.70 S—1.80 F	1.15	39.59	92.55	90.78	10.18
1.80 Sink	7.45	54.88	100.00	100.00	14.30

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 14.3 percent (laboratory no. C-2912, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.800 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.
[c] Sample specially handled in two size fractions, ½-inch by 3-mesh and 3-mesh by 20-mesh. Results separately plotted, then combined for present table.

TABLE 15.—WASHABILITY DATA[a] FOR COAL 8

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.284 Float	4.39	2.3	4.39	4.08	2.30
1.284 S—1.294 F	6.90	3.2	11.29	10.49	2.85
1.294 S—1.314 F	10.68	5.2	21.97	20.41	3.99
1.314 S—1.330 F	10.32	7.2	32.29	30.00	5.02
1.330 S—1.340 F	8.72	9.0	41.01	38.10	5.86
1.340 S—1.352 F	8.49	11.0	49.50	45.99	6.75
1.352 S—1.364 F	6.32	13.1	55.82	51.86	7.46
1.364 S—1.378 F	5.80	14.4	61.62	57.25	8.12
1.378 S—1.394 F	5.18	15.4	66.80	62.06	8 68
1.394 S—1.428 F	5.90	18.6	72.70	67.54	9.49
1.428 S—1.476 F	5.36	22.8	78.06	72.52	10.40
1.476 S—1.576 F	5.16	30.4	83.22	77.31	11.64
1.576 S—1.732 F	3.61	41.9	86.83	80.67	12.90
1.732 S—1.876 F	2.43	51.6	89.26	82.92	13.95
1.876 Sink	10.74	68.7	100.00	100.00	23.30

[a] Data are computed on dry basis.

[b] In order to make composited ash value agree with main head sample value of 23.3 percent (laboratory no. C-2932, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.876 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

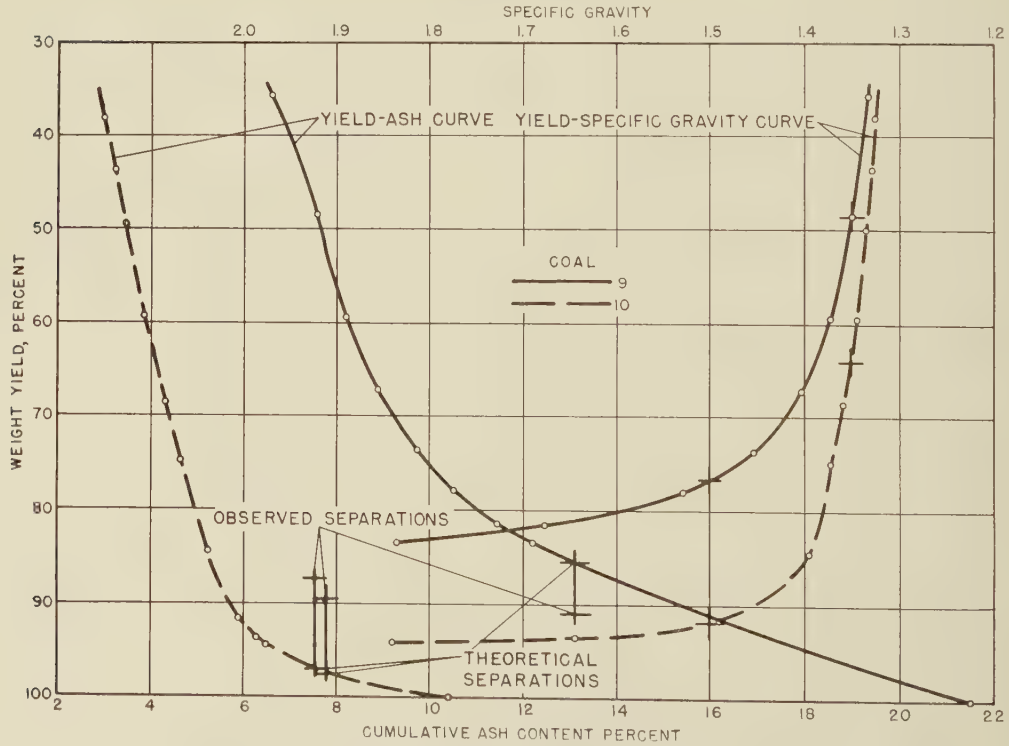

FIG. 6.—Yield-ash and yield-specific gravity curves for coals 9 and 10.

TABLE 16.—WASHABILITY DATA[a] FOR COAL 9

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.304 Float	14.74	4.0	14.74	13.47	4.00
1.304 S—1.320 F	12.92	6.5	27.66	25.28	5.17
1.320 S—1.332 F	11.55	10.1	39.21	35.83	6.62
1.332 S—1.350 F	13.95	10.3	53.16	48.58	7.59
1.350 S—1.372 F	11.94	11.1	65.10	59.49	8.23
1.372 S—1.404 F	8.42	14.4	73.52	67.18	8.94
1.404 S—1.454 F	7.05	18.2	80.57	73.63	9.75
1.454 S—1.530 F	4.81	23.3	85.38	78.02	10.51
1.530 S—1.676 F	3.98	31.5	89.36	81.66	11.45
1.676 S—1.835 F	2.06	46.6	91.42	83.54	12.24
1.835 Sink	8.58	68.5	100.00	100.00	21.50

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 21.5 percent (laboratory no. C-2953, table 2) the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.835 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 17.—WASHABILITY DATA[a] FOR COAL 10

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.310 Float	18.16	2.3	18.16	17.98	2.30
1.310 S—1.316 F	7.52	3.0	25.68	25.42	2.31
1.316 S—1.322 F	8.10	3.9	33.78	33.44	2.84
1.322 S—1.328 F	4.61	4.3	38.39	38.01	3.01
1.328 S—1.330 F	5.61	4.8	44.00	43.56	3.24
1.330 S—1.338 F	6.13	5.5	50.13	49.63	3.52
1.338 S—1.344 F	9.88	6.1	60.01	59.41	3.94
1.344 S—1.360 F	9.27	7.0	69.28	68.59	4.35
1.360 S—1.372 F	6.22	8.3	75.50	74.75	4.68
1.372 S—1.394 F	9.76	9.7	85.26	84.41	5.25
1.394 S—1.490 F	7.30	13.5	92.56	91.63	5.90
1.490 S—1.642 F	2.10	22.5	94.66	93.71	6.27
1.642 S—1.837 F	.62	35.1	95.28	94.33	6.46
1.837 Sink	4.72	75.9	100.00	100.00	10.40

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 10.4 percent (laboratory no. C-3024, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.837 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

Efficiency of separation (table 23) was computed in the manner most commonly used in coal washing, that is, as the ratio of actual yield obtained with the table to maximum theoretical yield of the same ash content, as determined from the yield-ash curves (figures 4 to 9). Efficiency so determined is well known to be theoretically defective in that zero actual separation does not result in an efficiency value of zero, as it logically should. Nevertheless, this measure of efficiency is useful for relative purposes, it is simply obtained if washability data are available, and it is commonly used. Several of the efficiency values are more than 100 percent despite every reasonable precaution in sampling.

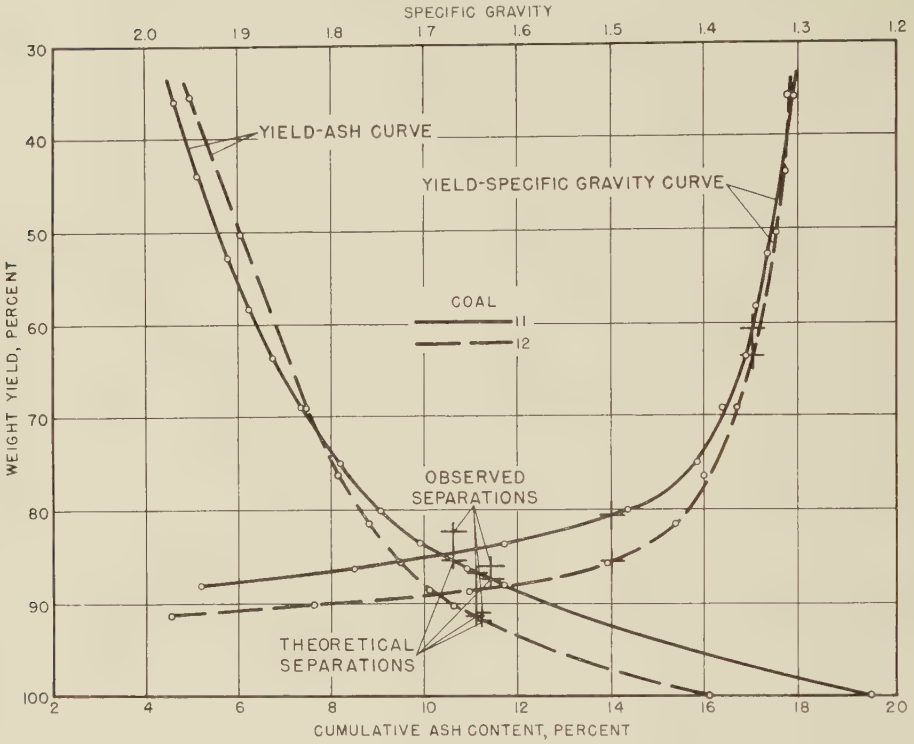

FIG. 7.—Yield-ash and yield-specific gravity curves for coals 11 and 12.

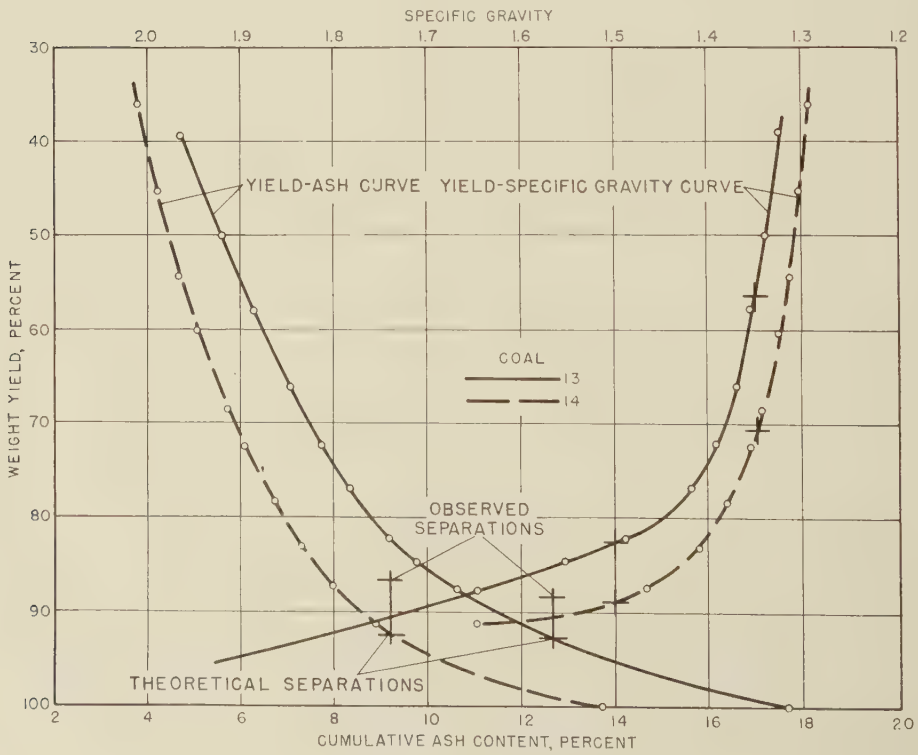

FIG. 8.—Yield-ash and yield-specific gravity curves for coals 13 and 14.

TABLE 18.—WASHABILITY DATA[a] FOR COAL 11

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.286 Float	22.33	3.9	22.33	22.38	3.90
1.286 S—1.306 F	13.58	5.9	35.91	35.99	4.66
1.306 S—1.316 F	7.97	7.3	43.88	43.98	5.14
1.316 S—1.335 F	8.87	9.1	52.75	52.87	5.80
1.335 S—1.346 F	5.32	10.3	58.07	58.20	6.21
1.346 S—1.358 F	5.41	12.9	63.48	63.63	6.78
1.358 S—1.382 F	5.46	13.9	68.94	69.10	7.35
1.382 S—1.410 F	5.82	18.3	74.76	74.93	8.20
1.410 S—1.484 F	5.13	21.3	79.89	80.07	9.04
1.484 S—1.614 F	3.47	29.3	83.36	83.55	9.89
1.614 S—1.776 F	2.63	42.5	85.99	86.19	10.88
1.776 S—1.940 F	1.86	51.0	87.85	88.05	11.73
1.940 Sink	12.15	76.7	100.00	100.00	19.50

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 19.5 percent (laboratory no. C-3079, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.940 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 19.—WASHABILITY DATA[a] FOR COAL 12

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.294 Float	17.23	3.6	17.23	16.92	3.60
1.294 S—1.312 F	19.03	6.2	36.26	35.61	4.96
1.312 S—1.325 F	15.01	8.7	51.27	50.36	6.06
1.325 S—1.366 F	19.12	11.3	70.39	69.14	7.48
1.366 S—1.400 F	7.20	15.1	77.59	76.21	8.19
1.400 S—1.430 F	5.36	18.2	82.95	81.47	8.84
1.430 S—1.502 F	4.52	22.3	87.47	85.91	9.53
1.502 S—1.650 F	2.91	28.4	90.38	88.77	10.14
1.650 S—1.816 F	1.46	42.2	91.84	90.21	10.65
1.816 S—1.970 F	1.12	48.8	92.96	91.31	11.11
1.970 Sink	7.04	68.6	100.00	100.00	16.11

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 16.1 percent (laboratory no. C-3132, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.970 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 20.—WASHABILITY DATA[a] FOR COAL 13

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.296 Float	16.66	3.2	16.66	16.20	3.20
1.296 S—1.312 F	13.38	5.0	30.04	29.20	4.00
1.312 S—1.324 F	10.45	6.9	40.49	39.36	4.75
1.324 S—1.340 F	11.02	8.8	51.51	50.07	5.61
1.340 S—1.356 F	8.15	10.8	59.66	58.00	6.32
1.356 S—1.371 F	8.30	12.6	67.96	66.06	7.09
1.371 S—1.394 F	6.38	15.0	74.34	72.27	7.77
1.394 S—1.420 F	4.73	17.2	79.07	76.86	8.33
1.420 S—1.490 F	5.43	21.4	84.50	82.14	9.17
1.490 S—1.554 F	2.60	28.3	87.10	84.67	9.74
1.554 S—1.649 F	3.20	35.5	90.12	87.61	10.61
1.649 Sink	9.88	67.0	100.00	100.00	17.60

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 17.6 percent (laboratory no. C-3204, table 2), cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.649 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 21.—WASHABILITY DATA[a] FOR COAL 14

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.274 Float	15.21	3.0	15.21	15.00	3.00
1.274 S—1.286 F	13.52	4.1	28.73	28.33	3.52
1.286 S—1.294 F	7.83	4.9	36.56	36.05	3.81
1.294 S—1.304 F	9.36	5.9	45.92	45.28	4.24
1.304 S—1.314 F	9.33	7.2	55.25	54.48	4.74
1.314 S—1.326 F	5.94	8.7	61.19	60.33	5.12
1.326 S—1.343 F	8.45	10.4	69.64	68.66	5.76
1.343 S—1.355 F	3.97	12.4	73.61	72.58	6.12
1.355 S—1.382 F	6.08	14.6	79.69	78.57	6.77
1.382 S—1.412 F	4.70	17.0	84.39	83.21	7.34
1.412 S—1.468 F	4.11	21.2	88.50	87.26	7.98
1.468 S—1.648 F	4.08	29.3	92.58	91.28	8.92
1.648 Sink	7.42	63.8	100.00	100.00	13.70

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 13.7 percent (laboratory no. C-3257, table 2), cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.648 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

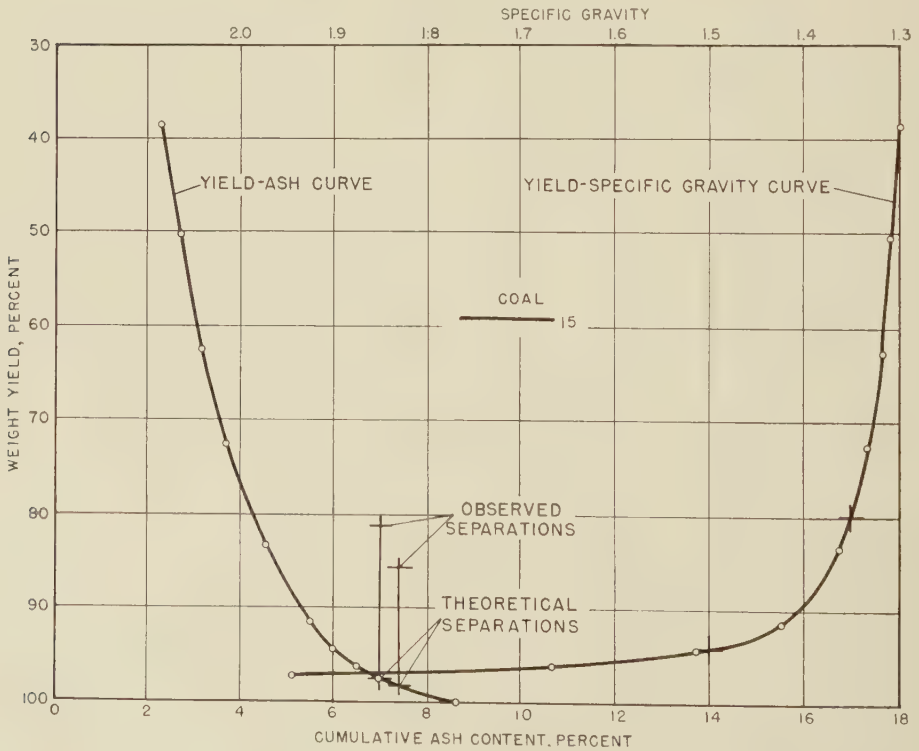

FIG. 9.—Yield-ash and yield-specific gravity curves for coal 15.

TABLE 22.—WASHABILITY DATA[a] FOR COAL 15

Specific gravity fractions	Fraction weight, percent of sample	Ash content, percent of fraction	Cumulative weight, percent of sample	Adjusted[b] cumulative weight, percent of sample	Cumulative ash content, percent of float
1.290 Float	20.41	1.8	20.41	20.38	1.80
1.290 S—1.298 F	17.88	2.9	38.29	38.23	2.31
1.298 S—1.308 F	12.08	4.0	50.37	50.28	2.72
1.308 S—1.316 F	12.24	5.1	62.61	62.50	3.18
1.316 S—1.334 F	10.20	6.9	72.81	72.69	3.70
1.334 S—1.362 F	10.57	10.4	83.38	83.24	4.55
1.362 S—1.424 F	8.27	14.9	91.65	91.49	5.49
1.424 S—1.514 F	2.87	21.3	94.52	94.36	5.97
1.514 S—1.668 F	1.82	34.7	96.34	96.18	6.51
1.668 S—1.945 F	1.13	47.0	97.47	97.30	6.98
1.945 Sink	2.53	67.2	100.00	100.00	8.60

[a] Data are computed on dry basis.
[b] In order to make composited ash value agree with main head sample value of 8.6 percent (laboratory no. C-3319, table 2), the cumulative weight column is adjusted by increasing the weight of the heaviest sink fraction (1.945 Sink) the necessary amount and then readjusting all weights to total 100 percent. See page 28.

TABLE 23.—EFFICIENCY OF SEPARATION

1	2	3	4	5
Run	Ash content of cleaned coal, percent	Weight yield		Efficiency of separation (ratio of col. 3 to col. 4), percent
		Actual, percent	Theoretical maximum,[a] percent	
51	9.8	90.9	85.5	106.3
52	9.4	88.8	84.8	104.7
61	10.6	92.6	87.4	105.9
62	11.1	95.2	88.9	107.1
71	10.7	92.1	92.4	99.7
72	10.0	75.6	90.2	83.8
81	15.0	89.9	85.0	105.8
82	12.9	79.4	80.6	98.5
91	13.1	91.1	85.5	106.5
101	7.6	87.4	97.0	90.1
102	7.8	89.5	97.4	91.9
111	11.4	86.0	87.4	98.4
112	10.6	82.3	85.5	96.3
121	11.1	86.8	91.3	95.1
122	11.2	91.1	91.8	99.2
131	12.6	88.3	92.6	95.4
141	9.2	86.8	92.5	93.8
151	7.4	85.7	98.3	87.2
152	7.0	81.1	97.5	83.2
161	8.9	81.8	[b]	[b]

[a] As observed on washability curves (figures 4 to 9), for weight yield corresponding to ash content of cleaned coal.
[b] Not obtained.

TABLE 24.—SIZE DATA

1	2	3	4	5	6	7	8	9	10	11	12
Coal number[a]	Head coal for run number	Washed coal from run number	Cumulative percentages by weight								Average size,[b] inches
			½″x 3-mesh	On 4-mesh	On 6-mesh	On 8-mesh	On 10-mesh	On 14-mesh	On 20-mesh	20-mesh x 0	
5A	51 and 52	.....	38.3	59.0	75.0	88.0	.....	97.7	98.2	1.8	0.240
5B	.....	51	32.5	54.7	72.5	86.1	95.0	97.5	98.4	1.6	0.226
5B′	.....	52	.....	.....	.....	(c)	.....	.....	.....	.....	.....
6A	61 and 62	.....	40.0	58.8	75.5	89.2	.....	98.0	98.3	1.7	0.243
6B	.....	61	.....	.....	.....	(c)	.....	.....	.....	.....	.....
6B′	.....	62	36.2	56.3	73.3	86.4	94.4	96.5	97.6	2.4	0.233
7A	71 and 72	.....	38.3	55.2	72.0	85.7	94.3	96.2	97.1	2.9	0.235
7B	.....	71	43.9	65.5	81.4	91.5	97.0	98.3	.....	.....	0.258
7C	.....	72	51.4	71.7	83.9	91.5	96.0	97.4	98.1	1.9	0.274
8A	81 and 82	.....	43.2	64.1	79.0	90.0	.....	97.1	97.7	2.3	0.253
8B	.....	81	52.4	72.8	85.8	93.9	98.0	98.8	99.2	0.8	0.279
8C	.....	82	52.7	74.0	85.8	93.4	97.5	98.4	98.9	1.1	0.280
9A	91	.....	42.0	61.5	77.2	89.6	.....	97.8	98.4	1.6	0.249
9B	.....	91	49.0	70.0	84.0	93.2	97.9	98.7	99.1	0.9	0.271
10A	101	.....	49.3	71.5	86.4	95.1	.....	98.9	99.1	0.9	0.274
10B	103	101	55.7	76.5	89.0	95.7	98.3	98.8	99.1	0.9	0.289
10C	.....	103	56.1	78.0	90.7	96.3	98.3	98.9	99.2	0.8	0.292
11A	111	.....	42.8	68.5	87.2	95.3	.....	98.5	98.7	1.3	0.262
11B	113	111	48.9	73.9	89.4	95.9	98.2	98.7	99.0	1.0	0.277
11C	.....	113	45.4	73.0	89.2	96.0	98.5	99.1	99.4	0.6	0.271
12A	121	.....	57.7	81.3	89.9	95.0	.....	98.5	98.7	1.3	0.295
12B	123	121	62.2	83.3	91.2	95.7	98.2	98.8	99.2	0.8	0.305
13A	131	.....	56.5	76.6	88.0	94.5	.....	98.7	99.0	1.0	0.289
13B	133	131	51.9	73.0	85.4	92.8	97.1	98.3	98.9	1.1	0.278
13C	.....	133	52.0	73.7	85.5	92.2	96.4	97.8	98.6	1.4	0.278
14A	141	.....	37.3	56.6	70.5	81.8	.....	92.3	92.8	7.2	0.229
14B	143	141	40.1	63.1	78.6	89.0	96.2	98.1	98.7	1.3	0.248
14C	.....	143	40.2	64.7	80.7	91.0	97.1	98.6	99.1	0.9	0.251
15A	151	.....	53.5	76.2	88.4	93.9	.....	98.3	98.7	1.3	0.284
15B	.....	151	55.2	77.6	88.5	93.8	97.0	98.0	98.6	1.4	0.287
16A	161	.....	55.6	80.2	89.7	94.9	98.0	98.7	99.0	1.0	0.291
16B	.....	161	63.7	84.8	93.4	96.7	98.5	99.1	99.4	0.6	0.310

[a] All coals with A suffixes were sized at ½-inch by 8-mesh in laboratory by means of vibrating screen.
[b] Weighted average linear opening of pairs of limiting screens.
[c] Data not obtained.

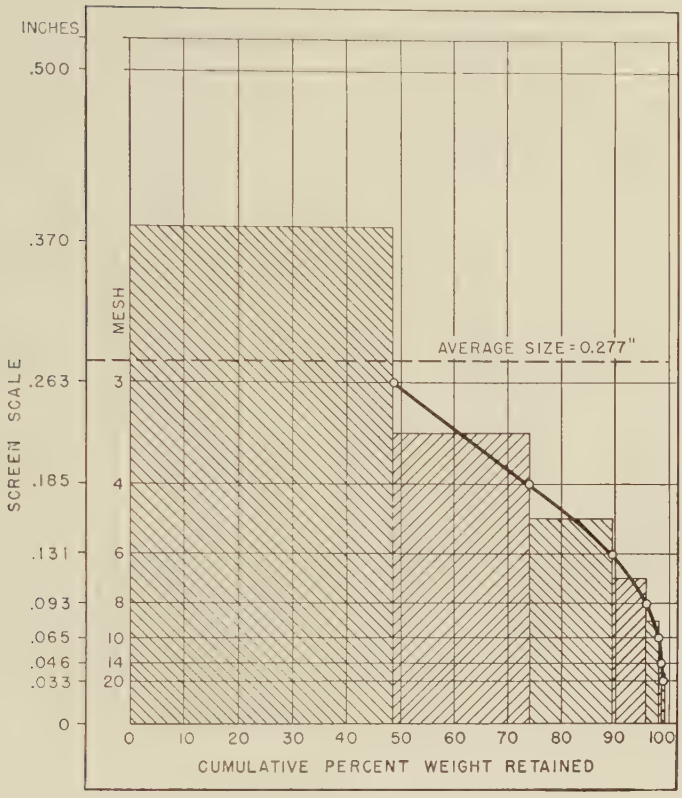

FIG. 10.—Illustration of computation of average size.
(Data from coal 11A, table 24.)

SIZE DATA

Size data were obtained on all raw coals and on all cleaned-coal products, with two exceptions in the earlier work. The data are presented in table 24.

Comparison is permitted by a computed average particle size (column 12 in table 24), representing the average of the linear openings of the various pairs of limiting screens used in the screening test, weighted in accordance with the percentages of coal in the various fractions.[30] Figure 10 indicates the manner of computation and the meaning of the result.

Average particle size for all raw coals was 0.260 inch, ranging from 0.229 to 0.295 inch (table 25), which was judged to be a satisfactory approach to constant size. The analysis for coal No. 14A appears so low as to be questionable. If it were omitted, average particle size for raw coals becomes 0.265 inch, ranging from 0.235 to 0.295 inch.

Table 25 also indicates a slight average increase in size associated with the tabling of the raw coals, amounting to an average of 0.013 inch (column 4). This average increase suggests that the average particle size of the material rejected by the table tends to be less than that of the raw coal. Although such a tendency may have been true for the tests herein reported, the present data do not permit any generalized statement of trend because it may easily be shown that the individual size changes (column 4 of table 25) vary too nearly randomly to make significant the observed average increase (standard deviation = 0.017 inch).

It was concluded that differences in size were too small to have appreciable effect on either combustion or tabling.

[30] For precedent in the use of this method of computation of average size, see Bird, Byron M. The sizing action of a coal-washing table. U. S. Bur. Mines RI 2755, 8 pp., 1926; p. 2; or Parry, V. F., and Goodman, John B. Briquetting subbituminous coal. U. S. Bur. Mines RI 3707, 37 pp., 1943; p. 11.

TABLE 25.—EFFECT OF TABLING ON AVERAGE SIZE

1	2	3	4	5	6	7	8
Tabling raw coals				Tabling washed coals			
Run No.	Average size[a] of raw coal, inches	Average size[a] of cleaned coal, inches	Change, inches	Run No	Average size[a] before retabling, inches	Average size[a] after retabling, inches	Change, inches
51	0.240	0.226	−0.014	103	0.289	0.292	+0.003
62	0.243	0.233	−0.010	113	0.277	0.271	−0.006
71	0.235	0.258	+0.023	133	0.278	0.278	0.000
72	0.235	0.274	+0.039	143	0.248	0.251	+0.003
81	0.253	0.279	+0.026				
82	0.253	0.280	+0.027	Av.	0.273	0.273	0.000
91	0.249	0.271	+0.022				
101	0.274	0.289	+0.015				
111	0.262	0.277	+0.015				
121	0.295	0.305	+0.010				
131	0.289	0.278	−0.011				
141	0.229	0.248	+0.019				
151	0.284	0.287	+0.003				
161	0.291	0.310	+0.019				
Av.	0.260	0.273	+0.013				

[a] Weighted average linear opening of pairs of limiting screens.

ZONE SAMPLES

Zone samples were taken along the washed coal edge of the concentrating table during the washing of five raw coals. Weight and dry ash data appear in table 26.

Although the composited ash contents of these sets agree closely with the ash content of the main samples of washed coal (a maximum difference of 0.4 percentage figure), there is no systematic increase in ash from the head-motion end excepting in run 161. Such an increase, when it occurs, makes possible lower yield and higher quality by simply moving the point of split between washed coal and reject toward the head-motion end. Theory suggests that a progressive increase in ash from the head-motion end is to be expected; but in four of the five sets of present data an ash content occurred within 28 inches of the head-motion end which was higher than the average ash content of the entire washed product.

It was concluded that zone-sample data did not contribute to an analysis of tabling results as related to the operating variables.

TABLE 26.—ZONE SAMPLE DATA

	Run 122	Run 131	Run 141	Run 151	Run 161
Zone 1[a]					
Weight, percent of washed coal	30.2	24.0	25.8	25.8	14.5
Ash, percent	9.8	9.2	10.4	5.7	6.5
Zone 2[b]					
Weight, percent of washed coal	33.3	39.0	46.2	22.9	22.2
Ash, percent	13.0	15.3	8.1	8.0	8.3
Zone 3[c]					
Weight, percent of washed coal	27.3	24.5	22.2	16.3	16.1
Ash, percent	10.7	11.5	8.7	7.3	9.0
Zone 4[d]					
Weight, percent of washed coal	9.2	12.5	5.8	35.0	47.2
Ash, percent	11.5	13.8	8.7	7.3	9.3
Ash Content					
Composited from zone samples, percent	11.2	12.7	8.9	7.0	8.6
Washed coal, percent	11.2	12.6	9.2	7.4	8.9

[a] Distance from head-motion end of table, 0 to 16 inches.
[b] Distance from head-motion end of table, 16 to 28 inches.
[c] Distance from head-motion end of table, 28 to 40 inches.
[d] Distance from head-motion end of table, 40 to 106 inches.

ANALYSIS OF INFLUENCE OF OPERATING VARIABLES ON TABLING PERFORMANCE

INTRODUCTION

It has been noted that direct evaluation of the effects of the variables upon quality changes, by means of many repeated tablings of a single coal, were judged to require much more time than could be conveniently arranged within the established laboratory schedule. Instead, each test was so conducted as to yield the best possible visual separation, all variables being adjusted to achieve this end.

The analysis of data arising in this manner is more difficult than that of data arising under fully controlled conditions, but applicable methods have been developed and are in common use by students of the behavior of living things.[31] Conclusions may be developed by such methods with great economies in the experimental plan. In the present study, they afford at least one advantage in that they apply to several coals, whereas an evaluation carried out as suggested in the last paragraph would, strictly speaking, have been applicable only to the single coal used. Further experimentation with other coals would then have been desirable.

Application of the methods for handling such experimentally uncontrolled data involves two assumptions: (1) The effects of each variable are essentially linear with the variable; and (2) the data available are reasonably representative of the whole possible population of like data. In line with these assumptions only the data obtained in substantially normal operation of the table were used. Tests involving extreme or abnormal settings were not included.

[31] Among such students are psychologists, dealing with individuals whose personal traits may, in general, differ uncontrollably, and agriculturists, dealing with the effects of various successive growing seasons in which rainfall, sunshine, and other climatic factors differ uncontrollably and in which data are often limited in number. Reference is made to:

Rider, Paul R. Modern statistical methods. Wiley (New York), 220 pp., 1939; Chapter IX, Experimental design.

Fisher, R. A. Statistical methods for research workers. Oliver and Boyd (London), 4th ed., 307 pp., 1932.

Fisher, R. A. Design of experiments. Oliver and Boyd (London), 252 pp., 1935.

Snedecor, George W. Statistical methods applied to experiments in agriculture and biology. Collegiate Press (Ames, Iowa), 341 pp., 1937.

Hagood, Margaret Jarman. Statistics for sociologists. Reynal and Hitchcock (New York), 934 pp., 1941.

TABLE 27.—TEST DATA USED IN ANALYSIS OF EFFECTS OF OPERATING VARIABLES

Item No.	Test No.	51	52	62	71	72
	Quality changes:					
1	Percentage increase in heating value	10.6	10.8	6.6	6.0	6.8
2	Percentage decrease in mineral matter	45.7	47.7	36.4	26.6	31.0
	Coal:					
3	Percentage of material of less than 1.35 specific gravity[b]	60.0	60.0	70.6	57.8	57.8
4	Percentage of material of greater than 1.50 specific gravity[b]	20.5	20.5	13.5	15.2	15.2
	Operating variables—primary:					
5	Rate of coal feed (lb./min.)	28.2	25.3	59.7	39.7	39.7
6	Rate of introduction of wash water (gal./min.)	41.4	42.6	46.9	38.9	40.5
7	Table stroke (in.)	1.25	1.25	.69	.69	.88
8	Speed of reciprocation of table (cycles/min.)	224	224	282	268	290
9	Transverse slope (deg.)	5.35	2.67	2.75	4.58	2.08
10	Longitudinal slope (deg.)	2.77	2.77	2.23	1.97	1.17
	Operating variables—secondary:					
11	Water/coal ratio	12.24	14.05	6.72	8.17	8.48
12	Table movement (in./min.)	560	560	388	469	508
13	Composite slope (deg.)	4.10	2.72	2.50	3.32	1.63
	Coal yield:					
14	Weight (percent of raw coal)	90.9	88.8	95.2	92.1	75.6
15	B.t.u. (percent recovered from raw coal)	100.5	98.4	101.5	97.6	80.7
16	Washing efficiency (percent)	106.3	104.7	107.1	99.7	83.8

[a] All data are on dry basis.
[b] From yield—specific gravity curves in figures 4 to 9.

DATA USED

The data employed, comprising eighteen tests, are set forth in table 27. (For each of two additional tests, one item of data is missing, requiring the rejection of the entire test as far as the present analysis is concerned.) All data are computed on a dry basis. Quality changes are evaluated by: (1) Percentage increase in heating value; and (2) percentage decrease in mineral matter (items 1 and 2, respectively). The nature of the specific gravity distribution of the coal feed is indicated by: (1) Percentage of material of less than 1.35 specific gravity; and (2) percentage of material of greater than 1.50 specific gravity (items 3 and 4, respectively). Size, the only other important coal characteristic affecting tabling operation, was substantially constant throughout. The six selected operating variables are indicated in table 27 (items 5 through 10), together with three combinations of them of possible importance: (1) The ratio of water to coal by weight; (2) composite angle of table slope, combining longitudinal and transverse slopes into a single figure equal to the slope of the table normal to the diagonal from feed to discharge corner; and (3) total deck movement in inches per minute (items 11, 12, and 13, respectively). Yield, both by weight and by recovered B.t.u., is also given (items 14 and 15, respectively) as is efficiency of separation (item 16).

It will be noted that for several samples the yield of B.t.u. slightly exceeds 100 percent, which is theoretically impossible. Sampling or analytical errors, or both, must be blamed, although great care was exercised at every stage in the work. The fact that such errors still creep in indicates the importance of regarding each reported figure merely as an approximation to the true but unknown value of the indicated characteristic.

ON PERFORMANCE OF CONCENTRATING TABLE AS REFLECTED BY QUALITY CHANGES[31]

81	82	91	101	102	111	112	121	122	131	141	151	152
10.7	13.9	11.5	3.3	3.0	11.7	12.9	5.8	6.2	6.1	5.6	0.8	1.9
34.8	43.4	38.5	26.3	24.4	39.3	43.4	28.3	27.7	27.7	29.5	13.5	17.3
44.5	44.5	48.6	64.0	64.0	60.8	60.8	63.4	63.4	56.6	70.5	79.8	79.8
26.0	26.0	23.4	8.2	8.2	19.4	19.4	14.4	14.4	17.5	11.0	5.8	5.8
29.5	33.7	38.1	53.4	38.6	35.4	21.9	45.2	35.8	37.4	28.0	31.1	35.6
29.5	33.7	37.4	30.4	30.4	32.7	33.8	32.5	33.9	35.1	25.9	21.7	21.8
.88	.88	.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.06	1.06
272	272	272	272	272	272	272	280	280	282	278	278	278
5.42	2.40	4.17	3.70	3.70	2.75	2.75	2.75	2.67	3.02	2.83	2.13	2.13
1.23	0.92	1.07	0.98	0.98	1.03	1.03	0.83	0.83	0.75	0.78	0.93	0.93
8.33	8.33	8.18	4.74	6.56	7.70	12.87	6.00	7.90	7.83	7.72	5.82	5.10
476	476	476	544	544	544	544	560	560	564	591	591	591
3.38	1.60	2.67	2.38	2.38	1.92	1.92	1.82	1.77	1.92	1.83	1.55	1.55
89.9	79.4	91.1	87.4	89.5	86.0	82.3	86.8	91.1	88.3	86.8	85.7	81.1
99.6	90.4	101.5	90.3	92.2	96.0	92.9	91.8	96.8	93.7	91.7	86.4	82.6
105.8	98.5	106.5	90.1	91.9	98.4	96.3	95.1	99.2	95.4	93.8	87.2	83.2

CORRELATION COEFFICIENTS

The degree of relationship between each of the items in table 27 with each other item is expressed in terms of correlation coefficients[32] in table 28.

A correlation coefficient of zero indicates complete absence of relationship between two variables, and a correlation coefficient of plus or minus one indicates perfect relationship, such as would yield an exactly straight line if all values of the variables were plotted. All correlation coefficients fall between plus one and minus one, positive coefficients indicating that large values of one variable are associated with large values of the other variable, and negative coefficients indicating that large values of one variable are associated with small values of the other variable. A measure of the strength of relationship between the variables is given by the numerical value of the correlation coefficient, regardless of sign. Coefficients of exactly plus or minus one are almost never found in laboratory data because of the effect of other variables and of observational errors. It is assumed that the latter are randomly distributed and not due to assignable causes, such as an improperly adjusted balance reading consistently high or low. For similar reasons, a correlation coefficient of exactly zero is almost never encountered.

For purposes of visualization, correlations yielding coefficients of approximately 0.1, 0.5, and 0.9 are illustrated in figures 11, 12, and 13, respectively.

From a statistical viewpoint, any data at hand can be considered as a sample of a very large mass of data which could arise from prolonged repetition of the same type of experiment. Although the only information available on such a theoretically conceivable mass of data (usually called a "population") is the data at hand, interest basically centers in the population; and it is desired to gain as much information as possible about the population from the available sample of data. It is further desired that an estimate be formed of the probable accuracy of any information deduced.

[32] For method of computation, see Appendix A.

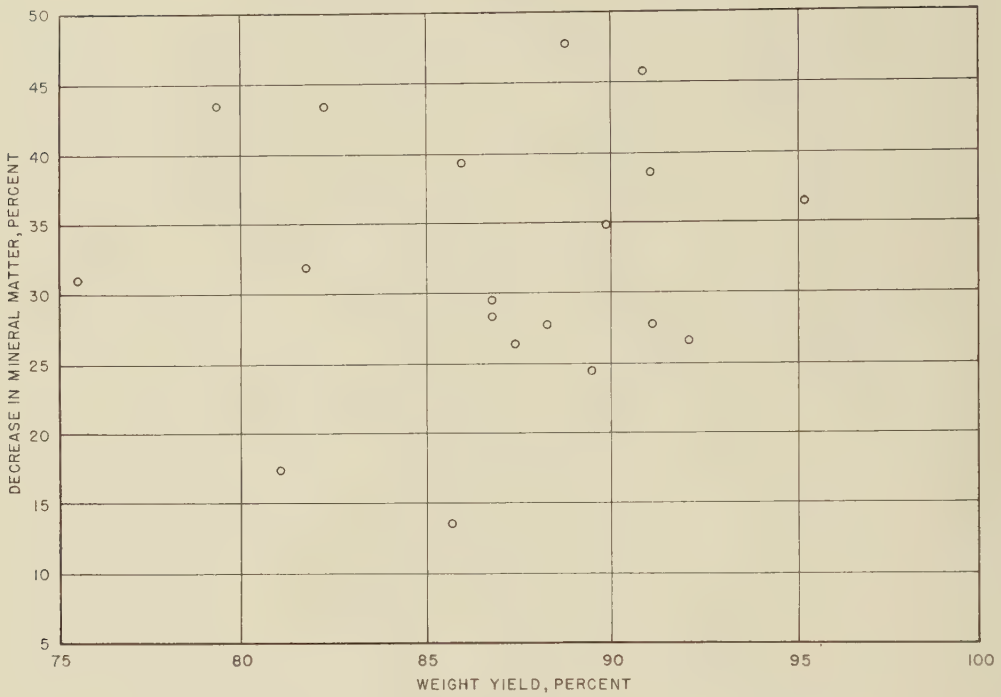

FIG. 11.—Relationship of decrease in mineral matter to weight yield.
$r = +0.070$

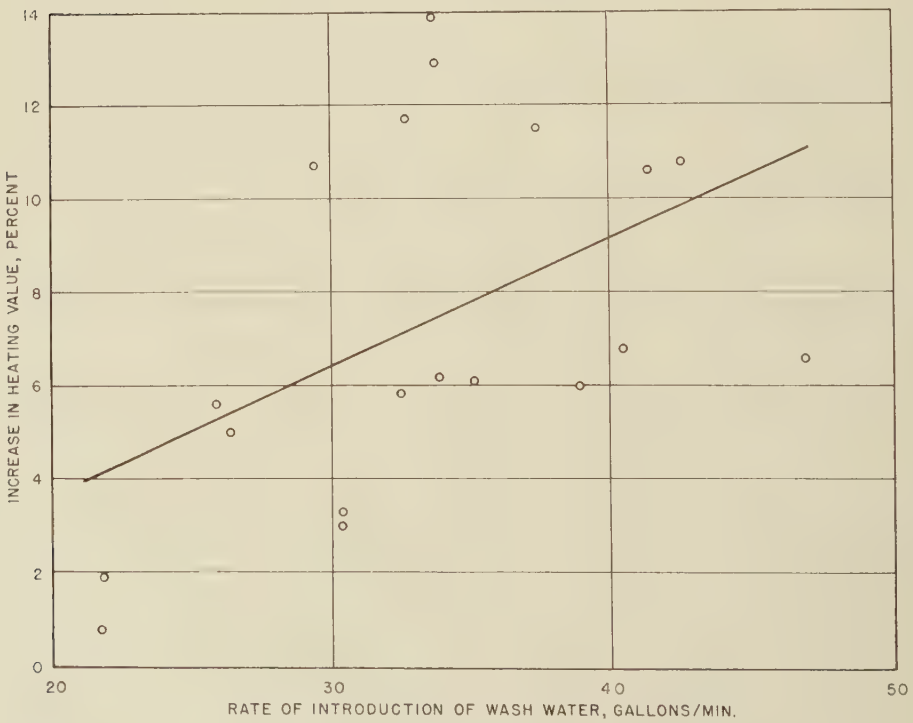

FIG. 12.—Relationship of increase in heating value to rate of introduction of wash water.
$r = +0.477$

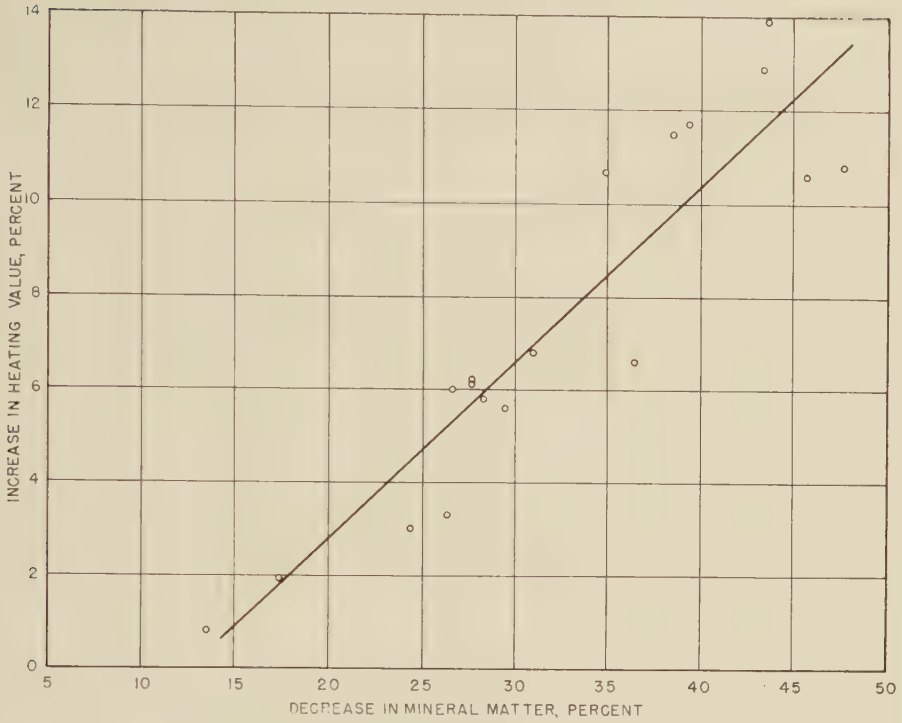

FIG. 13.—Relationship of increase in heating value to decrease in mineral matter. $r = 0.920$.

Clearly, in any sampling from a population of data, there is always some possibility that the particular data forming the sample could, by chance, show an apparent correlation when none actually exists in the population. The probability of a sample thus exhibiting a correlation when the population is actually uncorrelated is less if the sample is large and as the magnitude of the observed correlation becomes larger.

It will be observed in figure 11 that a correlation coefficient of approximately 0.1 (more exactly, 0.070) is indicative of almost complete lack of association between the variables. In the case of these data, the probability discussed above is approximately 0.77;[33] that is, from the sample of 18 pairs of data at hand, it may be inferred that other samples of 18 drawn at random from data in which there was no correlation whatsoever would show correlations as great as 0.070 approximately 77 times out of 100. Hence, such a correlation in samples of 18 units of data indicates almost certain absence of true relationship between the variables.

Samples of 18 pairs of data from other variables may yield numerically larger correlation coefficients, with rapidly increasing probabilities that the variables are actually related. When a coefficient of 0.468 (either positive or negative) is reached, the probability is only 0.05 that the observed data might have arisen from an uncorrelated population of data.[34] At this point it is common to place a "reasonable" confidence in the existence of a true trend, and the correlation coefficient is sometimes said to be "significant" in accordance with that standard of probability. In certain fields of work more rigorous or less rigorous standards may be observed.[35]

[33] For method of computation, see Rider, Paul R. Op. cit.; p. 83.

[34] Snedecor, George W. Op. cit.; p. 125.

[35] "It should perhaps be emphasized that 'significance' is a relative term. Thus, one person might regard a deviation as significant if the probability of the occurrence of a greater deviation were 0.05. Another might regard it as significant only if this probability were 0.001. It is largely a subjective matter and depends upon the chances that the individual is willing to take that his judgment may be wrong. Many investigators are willing to regard as *significant* any deviation (or difference) for which the probability of a greater deviation is 0.05, and as *highly significant* any deviation for which this probability is 0.01 or less." Rider, Paul R. Op. cit.; p. 78.

TABLE 28.—CORRELATION COEFFICIENTS[a]

	Percentage decrease in mineral matter	Rate of coal feed	Rate of introduction of wash water	Table stroke	Speed of reciprocation of table
Percentage increase in heating value	+.920	−.393	+.477	−.040	−.378
Percentage decrease in mineral matter		−.285	+.661	+.102	−.582
Rate of coal feed			+.263	−.601	+.428
Rate of introduction of wash water				−.207	−.366
Table stroke					−.708
Speed of reciprocation of table					
Transverse slope					
Longitudinal slope					
Water coal ratio					
Table movement					
Composite slope					
Material of specific gravity less than 1.35					
Material of specific gravity greater than 1.50					
Weight yield					
B.t.u. yield					

[a] Appendix A.

Figure 12 illustrates a correlation of +0.477, just over the border-line of significance. It will be observed that the data are still widely scattered, but chances are slightly better than 95 out of 100 that a trend exists. The trend line best fitting the present data is shown. The acquisition of more data would be likely to influence the trend line materially.

A correlation coefficient of +0.920 is illustrated in figure 13. Although there appears to be no doubt of the association between percentage increase in heating value and percentage decrease in mineral matter, a statistician would cautiously state that the probability is very high (well over 0.99) that a relationship exists. Further data might affect the indicated trend line in this figure also, but to a lesser extent than that in figure 12.

The degree of confidence which may be placed in any given correlation coefficient, as measured by the above-discussed probability of existence of true trends, is influenced both by the quantity of data and by the number of variables involved. The latter factor is of no concern for the correlations in table 28, but is of importance in studying partial[36] or multiple correlations. With reference to the influence of quantity of data, it is fairly evident that a correlation coefficient indicated by a few data is much more likely to be fortuitous than is one of equal numerical magnitude indicated by ten times as many data, all determined with equal care. It has been shown by small-sample theory in mathematical statistics[37] that a correlation coefficient of 0.361 for thirty pairs of data, and of only 0.197 for 100 pairs of data, are sufficient to make

[36] See discussion of partial correlation coefficients, p. 49.
[37] Snedecor, George W. Op. cit.; table 7.2, p. 125.

COMPUTED FROM DATA IN TABLE 27

Transverse slope	Longitudinal slope	Water/coal ratio	Table movement	Composite slope	Material of specific gravity less than 1.35	Material of specific gravity greater than 1.50	Weight yield	B.t.u. yield	Efficiency
+.241	+.270	+.695	−.359	+.287	−.750	+.934	−.058	+.496	+.653
+.249	+.527	+.789	−.341	+.410	−.609	+.816	+.070	+.564	+.707
−.063	−.033	−.648	−.497	−.059	+.153	−.339	+.278	+.020	−.049
+.210	+.698	+.522	−.612	+.464	−.400	+.479	+.329	+.545	+.615
+.033	+.257	+.435	+.812	+.144	+.255	−.114	−.064	−.069	−.096
−.424	−.798	−.719	−.157	−.668	+.147	−.333	−.280	−.457	−.499
.	+.375	+.197	−.274	+.910	−.481	+.387	+.567	+.630	+.574
.	.	+.599	−.303	+.724	−.053	+.254	+.425	+.522	+.575
.	.	.	+.011	+.410	−.367	+.603	+.016	+.398	+.501
.	.	.	.	−.330	+.467	−.428	−.299	−.451	−.530
.	.	.	.	.	−.367	+.387	+.619	+.701	+.677
.	.	.	.	.	.	−.883	−.006	−.413	−.530
.	.	.	.	.	.	.	+.042	+.549	+.708
.	.	.	.	.	.	.	.	+.838	+.693
.	.	.	.	.	.	.	.	.	+.960

reasonably sure of the existence of a trend, in accordance with the standard of a 0.95 probability. These are to be compared with 0.468 for 18 pairs, as are here available.

Correlations numerically less than 0.468 are not necessarily to be ignored. They are simply too small to establish beyond a reasonable doubt of stated magnitude that they represent true trends, rather than sampling fluctuations, in the population from which the available data are conceived to be drawn as a sample. It should be remembered that the true value of the correlation coefficient in the population is almost as likely to be greater than the sample value as it is to be less.[38] Furthermore, if more data exhibiting the same correlation became available, the confidence which may be placed in the indicated trend would be increased, possibly to the established level of significance.

In effect, table 28 permits rapid and objective comparison of 120 pairings of variables which would otherwise require 120 separate plots. The advantages of objectivity and condensation are gained, however, at the sacrifice of two advantages possessed by numerous individual plots: (1) Detection of curvilinear trends; and (2) detection of individually erratic units of data. With regard to (1), it may be said that a great many plots, involving the data in table 27 and numerous other data, have been made and no curvilinear trends of any importance were noted. In the course of these, certain erratic data have been detected, as suggested in (2), giving rise to further avenues of analysis.

[38] Wallace, H. A., and Snedecor, George W. Correlation and machine calculation. Iowa State College, 71 pp., 1931; p. 64.

TABLE 29.—SIGNIFICANT CORRELATION COEFFICIENTS BETWEEN OPERATING VARIABLES AND MEASURES OF PERFORMANCE, FROM TABLE 28[a]

	1	2	3	4	5
Operating variables	Measures of performance: Percentage increase in heating value	Percentage decrease in mineral matter	B.t.u. yield	Efficiency	Average
Rate of introduction of wash water	+.477*	+.661*	+.545*	+.615*	+.575
Water/coal ratio	+.695*	+.789*	+.393	+.501*	+.596
Speed of reciprocation of table	−.378	−.582*	−.457	−.499*	−.479
Longitudinal slope	+.270	+.527*	+.522*	+.575*	+.474
Transverse slope	+.241	+.249	+.630*	+.574*	+.424
Composite slope	+.287	+.410	+.701*	+.677*	+.677
Table movement	−.359	−.341	−.451	−.530*	−.420

[a] A significant correlation coefficient is defined as one of such an absolute value that the probability of its arising by chance in a sample from an uncorrelated population is less than some assigned value, taken to be 0.05 for the present purposes. For a probability of 0.05 and for 18 pairs of data, the necessary absolute value is 0.468. See also page 45.
* Probability of true trend exceeds 0.95 ($|r| > 0.468$).

Significant correlation coefficients between operating variables and measures of performance are abstracted from table 28 and shown in a more convenient form in table 29. From this table it will be observed that rate of introduction of wash water and water/coal ratio are both significantly related to the two measures of quality improvement, *percentage increase in heating value* and *percentage decrease in mineral matter*. Furthermore, speed of reciprocation of the table and longitudinal slope appear to bear a significant association with percentage decrease in mineral matter. For *percentage recovery of heating units (B.t.u. yield)*, which is in a way another measure of concentrating table performance, the significantly related operating variables appear to be rate of introduction of wash water, longitudinal slope, transverse slope, and composite slope. *Efficiency* is seen to bear a significant relation to each of the operating variables mentioned, and in addition is significantly related to table movement.

As a rough measure of the relative influence of the variables on performance, the four correlation coefficients for each variable are averaged in column 5 of table 29. So computed, the importance of ample wash water is brought out by relatively high average coefficients for both rate of introduction of wash water and water/coal ratio. The slopes—longitudinal, transverse, and composite—appear to be of importance, although less so for the measures of performance relating to quality (percentage increase in heating value and percentage decrease in mineral matter) and more so for the measures relating to quantity (B.t.u. yield and efficiency). Speed of reciprocation has a fairly consistent negative correlation —that is, slower rate tends toward better performance, in the range of data obtained.

EFFECT OF OTHER VARIABLES

At this point it should be recognized that the correlation between any pair of variables indicated by table 28 makes no allowance for the possible influence of a third or other variables. Apparently correlated variables may actually bear on a third variable in such a way as to partly or entirely account for the apparent correlation. If the third variable could be allowed for, the true association between the apparently correlated variables might be found quite unimportant. An example may serve to make the idea clearer.

Consider the records over the years of two crops in a given region, and assume that these exhibit a relationship which is fairly strong—that is, assume that large yields

of one crop are fairly definitely associated with large yields of the other. Is the apparent association between yields of the two crops due to some underlying relation between these variables partaking of the nature of cause and effect, or is it merely due to a close association between each of them and rainfall, or sunshine, or temperature, or some combination of any or all of these which affect both crops similarly? By the appropriate methods, the degree of common association which each crop bears to any other known variables may be allowed for, within the limits of the data available, and the net tendency of large yields of one crop to be associated with large yields of the other crop, independent of the effects of the other designated variables, may be expressed. Such a net relationship would presumably be quite low, since it is not likely that any basic relation exists whereby the yield of one crop directly affects another, independent of growing conditions which influence them both.

Other situations exist in which a very poor correlation apparently exists between two variables, although logically the variables seem to be related. When the influence of a third variable is allowed for by the methods of partial correlation, the relationship between the two comes out in its true strength.

Partial Correlation Coefficients

The ability to make such allowances systematically and objectively constitutes a third advantage of an analysis of data by the methods of correlation. The measure used to evaluate net relationships between two variables, eliminating any portion of the apparent relationship between the two which may actually be a consequence of one or more other variables, is known as the partial correlation coefficient.[39]

For the present data, it is desired to determine the effects, if any, which the several operating variables individually have on performance, making proper allowance for variations from test to test in major non-operational variables. Selected as being major non-operational variables were weight yield and the nature of the raw coal tested, where the latter is defined in terms of specific gravity (the physical property of greatest importance in most coal cleaning) by using "percentage of raw material of less than 1.35 specific gravity" and "percentage of raw material of greater than 1.50 specific gravity."

Partial correlation coefficients, relating each measure of quality improvement or table performance with each of the operating variables, independent of the variations in specific gravity distribution of coals tested and in weight yield, are set out in table 30.

Owing to the increase in the number of variables simultaneously under consideration, the numerical value of a partial correlation coefficient for any given confidence level is slightly higher than that of a simple correlation coefficient for the same confidence level. For 18 units of data, a partial correlation coefficient relating two variables independent of changes in three others should be at least 0.514 to correspond to the confidence placed in a simple correlation coefficient of 0.468.[40]

Influence of Variables on Percentage Increase in Heating Value

Table 30 indicates that the effect of rate of introduction of wash water and of water/coal ratio on percentage increase in heating value is appreciably reduced when allowances are made for varying coals and varying weight yields ($r = +0.477$ reduced to $r = +0.222$, and $r = +0.695$ reduced to $r = +0.332$, respectively). The small positive values remaining are insufficient to warrant confidence that the trends are real and not due to sampling.

On the other hand, table 30 suggests the importance of another operating variable with regard to quality improvement—speed of reciprocation of the table. This is not commonly regarded as important as certain other variables, or perhaps it should be said that it is less commonly experimented with, possibly because speed-changing devices are

[39] For method of computation, see Appendix B.

[40] Wallace, H. A., and Snedecor, George W. Op. cit.; table 16, p. 62.

Table 30. Partial Correlation Coefficients Relating Measures of Performance to Operating Variables, Independent of Specific Gravity Distribution of Raw Coal[a] and of Weight Yield

Operating variables	Percentage increase in heating value	Percentage decrease in mineral matter	B.t.u. yield	Efficiency
Rate of coal feed	+.081	+.134	−.003	+.342
Rate of introduction of wash water	+.222	+.584*	+.210	+.450
Table stroke	+.071	+.242	+.066	−.175
Speed of reciprocation of table	−.544*	−.515*	−.214	−.254
Transverse slope	−.058	−.024	+.008	−.108
Longitudinal slope	+.072	+.530*	+.092	+.454
Water/coal ratio	+.332	+.569*	+.353	+.178
Table movement	−.085	−.070	−.049	−.491
Composite table slope	−.002	+.257	+.057	+.146

[a] Defined by percentage of raw material of less than 1.35 specific gravity and by percentage of raw material of greater than 1.50 specific gravity.
* Probability of true trend exceeds 0.95 ($|r| > 0.514$).

almost never included in a table installation. In table 28, the simple correlation of speed of reciprocation with percentage of increase of heating value is −0.378, indicating a negative trend but still below the 0.95 level of significance. However, by eliminating the effects of varying raw coals and of varying weight yields, the correlation is increased numerically to −0.544. Such a negative correlation may be interpreted to mean that if tests were run with the same coal, systematically varying the speed of reciprocation and readjusting any other operating variables as needed in order to maintain weight yield constant, and further, that if this procedure were to be repeated with a variety of coals and for a variety of weight yields over the range of such variables covered by the available data, the net effect of decreased speed of reciprocation would be in the direction of increased heating value in the cleaned coal.

The simple correlation coefficients between percentage increase in heating value and the variables for which allowance is made in table 30 should also be noted in table 28. "Percentage of material in feed of less than 1.35 specific gravity" (which may be referred to as "coal" for brevity) and "percentage of material in feed of greater than 1.50 specific gravity" (which may be referred to as "non-coal") both exhibit significant correlations with percentage increase in heating value, as is to be expected. For "coal," the value of $r = -0.750$ states that the more low-gravity material in the raw feed, the less percentage increase is to be gained in the cleaned coal, without regard to the influence of other factors. Similarly, for "non-coal," the value of $r = +0.934$ states that when a feed contains a large percentage of high-gravity material, a substantial percentage increase in heating value is probable.

It had been expected that weight yield would also show a numerically significant negative correlation with percentage increase in heating value, because common experience is that quality of cleaned coal decreases with increased weight yield. Table 28 shows a practically non-existent correlation, $r = -0.058$. Inasmuch as this does not allow for variations in coal, the question arises, how are weight yield and percentage increase in heating value related, making allowance for variations in coal (specifically in "coal" and "non-coal" as used in the preceding paragraph)? This may be computed as $r = -0.336$, which is not as high as might have been expected but is in the right direction.

Influence of Variables on Percentage Decrease in Mineral Matter

Passing now to the second criterion of improvement due to washing, "percentage decrease in mineral matter," it may first be of interest to observe that this criterion is related closely to the first ($r = +0.920$,

from table 28), as is to be expected. There is therefore reason to expect similarity in the relationships exhibited by the operating variables with each of the two measures of quality improvement.

Table 28 shows that the operating variables are associated more strongly with percentage decrease in mineral matter than with percentage increase in heating value. This may be because reduction in mineral matter is more nearly the direct result of coal cleaning than is increase in heating value; the latter is almost entirely a consequence of reduction of the noncombustible diluent.

When allowance is made for varying raw coals and varying weight yields, table 30 shows that the significant correlations observed in table 28 between percentage decrease in mineral matter and four operating variables are affected as set forth in table 31. The results definitely suggest that ample wash water is desirable for greatest improvement in quality, independent of changes in weight yield. The effect of speed of reciprocation, noted before, is substantiated; and it appears that increased longitudinal slope is conducive to cleaner coal, other operating variables being adjusted to maintain weight yield constant.

These indications that speed of reciprocation and longitudinal slope are important in producing the cleanest coal, when weight yield must be maintained for economic reasons, are particularly interesting in view of the fact that they are the two variables least commonly controlled. Arrangements for their adjustment during operation are unknown to the author in any installation, for either commercial or experimental purposes, except the laboratory in which these data were assembled.

Table 28 also shows that percentages of "coal" and "non-coal" in the raw coal feed affect the percentage decrease in mineral matter in much the same manner that they were observed to affect percentage increase in heating value. The correlation coefficients involved make no allowance for weight yield, but in view of the extremely small association which weight yield bears with the variables under consideration (good

Table 31.—Correlation Coefficients Between Percentage Decrease in Mineral Matter and Certain Operating Variables

Operating Variable	Without allowance for coal and for weight yield (from Table 28)	Allowing for coal and for weight yield (from Table 30)
Rate of introduction of wash water	+0.661	+0.584
Water/coal ratio	+0.789	+0.569
Speed of reciprocation of table	−0.582	−0.515
Longitudinal slope	+0.527	+0.530

"canceling out"), little change would result in making such allowance.

The lack of association between weight yield and percentage reduction in mineral matter ($r = +0.070$, table 28) corresponds to a similar lack of association previously noted between weight yield and percentage increase in heating value ($r = -0.058$, table 28). Neither of these allows for varying specific gravity distribution of the coal feed, but on page 50 a calculation making such allowance is reported, resulting in a numerical increase of the latter correlation to $r = -0.336$. A similar calculation, separating out the influence of specific gravity distribution from the apparent relationship between weight yield and percentage reduction in mineral matter, yields $r = +0.039$. The reason for this almost complete lack of association is not known. A fairly strong negative correlation coefficient was expected, in accordance with the general observation that increased weight yield reduces the percentage of reduction of mineral matter for any given coal.

Influence of Variables on Yield of Heat Units

Owing to the economic importance of percentage recovery of heat units in the cleaned coal, it was considered desirable to evaluate effects on it of several operating variables, independent of the specific gravity distribution of the raw coal and of weight yield. Percentage recovery of heat units in

the cleaned coal, or B.t.u. yield, is equivalent to the weight yield of cleaned coal multiplied by the ratio of the heating value of cleaned coal to that of the raw coal. In a sense, it is a measure of the economy of a cleaning process, although theoretically it will go to a maximum of complete recovery of heating units only when there is no reject and consequently no quality improvement in the cleaned coal.[41]

Table 30 indicates no significantly strong relation between B.t.u. yield and the operating variables, after allowing for the influences of kind of coal (as defined by "coal" and "non-coal") and of weight yield. This might not have been expected from the simple correlation coefficients in table 28, where several operating variables appear to correlate fairly well with B.t.u. yield, notably transverse slope and composite slope. The partial correlation coefficients of table 30 show that the apparent correlations in table 28 are due for the most part to common associations with other variables. This may be taken to mean that in repeated tablings of the same coal, at the same weight yield, systematic changes of any one of the operating variables (with adjustments of the remaining operating variables as need be to maintain constant weight yield) are not likely systematically to influence B.t.u. yield. From another viewpoint, it may be said that a necessary or convenient change in one of the operating variables (rate of coal feed, for example) can be compensated for as a general thing by appropriate changes in the other variables, so as to maintain B.t.u. yield substantially constant.

The last two sentences are general statements and do not exclude the possibility that instances may occur where a variation of some one operating variable, for some coal at some weight yield, will systematically influence B.t.u. yield. In general, however, such instances are not to be expected.

Influence of Variables on Efficiency

When strict attention is paid to a minimum level of significance of 0.95, requiring a partial correlation coefficient numerically equal to at least 0.514 for 18 sets of data, table 30 indicates that the effect of none of the variables on efficiency of separation can be stated with confidence. However, three of the variables—rate of introduction of wash water, longitudinal slope, and table movement—fall only slightly short of the minimum and might with more data be established as significant.

Bearing in mind the qualification that the correlations are lower than desired, rate of introduction of wash water and longitudinal slope are positively related to efficiency. Increases in either of them, with weight yield constant, would in general increase efficiency of separation, within the range of the present data. Table movement is negatively correlated—that is, increased table movement tends to be detrimental to maximum efficiency.

The evidence on the effect of table movement is opposite to that presented by Yancey and Black[42] for a single Colorado coal approximately 3-mesh by zero in size on a full-sized table. Whether double-screened stoker coal of larger size (nominally ½-inch by 8-mesh) on a half-sized table actually behaves differently than the coal used by Yancey and Black, or the admittedly low correlation between table movement and efficiency (table 30) should not be considered, is not known. It is clear that further experimentation on the subject is needed.

Summary of Analysis of Effect of Variables on Performance

The data indicate that increased wash water, increased water/coal ratio, increased longitudinal slope, and decreased speed of reciprocation are conducive to improved results under conditions of constant weight yield, when *percentage decrease in mineral matter* is accepted as the criterion of movement. When *percentage increase in heating value* is used to measure improvement in quality, decreased speed of reciprocation is shown to be important, with the correlations with other variables being at lower

[41] See discussion of data used, page 42, for remarks about values of B.t.u. yield exceeding 100 in table 27.

[42] Yancey, H. F., and Black, C. G. Op. cit.

levels. Little or no tendency to affect quality of results, independent of weight yield, is shown by the data for rate of coal feed, transverse slope, table stroke, table movement, or composite slope. *Percentage recovery of heat units (B.t.u. yield)* does not appear to be systematically influenced by any of the operating variables, independent of weight yield. There is some evidence that *efficiency of separation* tends to be increased by more wash water, by greater longitudinal slope, and by reduced rate of table movement.

The number of tests is not as large as might be desirable, but the above-mentioned tendencies are strong enough to warrant reasonable confidence in them, within the range of the data obtained.

ECONOMICS OF COAL WASHING, WITH PARTICULAR REFERENCE TO THE CONCENTRATING TABLE

Washed coal produced by a concentrating table is suited by its size primarily to two major markets, metallurgical and stoker. The advantages of low ash and low sulfur content in coal for metallurgical coke are so pronounced and carry so far throughout the entire procedure of iron and steel making that very substantial expense for coal washing is justified. For this reason the steel industry has been a leader in advocating better coal-washing methods.[43]

The economic advantages of washing coal for steam and domestic purposes are not usually so clear. The present discussion is concerned with the washing of coal for domestic use, with particular reference to washed stoker coal produced by the concentrating table. An attempt is made to consider the major factors by which relative desirability of washed and raw coal for domestic use are judged, these being regarded as cost, convenience, and performance. The merits of the concentrating table as a coal washing device are also considered.

[43] Yancey, H. F., and Fraser, Thomas. Op. cit., pp. 15–20.

Cost

Delivered cost, or cost to the consumer, is considered to reflect cost of production and loading plus cost of transportation. Dealers' margins and pricing policies are disregarded as subject to local competitive variations.

COST OF PRODUCTION

It is always more expensive to produce washed coal, per unit of potential heat, than unwashed coal because any washing process discards some combustible, leaving a reduced amount of combustible over which to spread mining costs plus the cost of the washing process itself.

To illustrate the increase in cost, consider a separation with a weight yield of 90 percent, and with a heat-unit recovery of 95 percent in the washed coal, which ordinarily would be regarded as quite satisfactory. On an assumed cost basis of $1.50 per ton to mine and to load raw coal, and of 10 cents per ton for the washing process, washed coal will cost $1.78 per ton to produce and to load, an increase of 18.7 percent.

Under such conditions, the heating value of the washed coal will be about 5.6 percent higher than that of the unwashed coal, less than one-third of the percentage increase in cost. Allowing for the increase in heating value of the washed coal, its increase in cost per unit of potential heat is about 12.3 percent.

Recent changes in wage rates may make the assumed cost data somewhat low, but the point that washed coal costs more than raw coal to produce, per unit of potential heat, is still clear.

Table 32 gives corresponding data for the coals washed in the present investigation, making the same assumptions of $1.50 and 10 cents for mining cost and for processing cost, respectively.

INCREASE IN EFFICIENCY

The question then arises: Will improved efficiency of combustion of washed coal compensate for the increased costs assignable to the washing process?

TABLE 32.—RELATIVE COSTS OF PRODUCTION OF RAW AND CLEANED

1	Cleaning run number	51	52	61	62	17
2	Moisture in raw coal, as received basis	12.6	12.6	14.3	14.3	2.8
3	Cost to mine and load raw coal, cents/ton[a]	150	150	150	150	150
4	Cost to mine and load raw coal, cents/million B.t.u., as-rec'd basis	7.54	7.54	7.43	7.43	6.10
5	Weight yield of washed coal, percent[b]	90.9	88.8	92.6	95.2	92.1
6	Cost to mine, clean, and load cleaned coal, cents/ton[b,c]	176	180	173	168	174
7	Increase of production cost of cleaned over raw coal, cents/ton	26	30	23	18	24
8	Increase of production cost of cleaned over raw coal, percent/ton	17.3	20.1	15.2	12.0	15.8
9	Increase of heating value in cleaned coal, percent[b,d]	10.6	10.8	8.4	6.6	6.0
10	B.t.u. recovery in cleaned coal, percent[b,d]	100.5	98.4	100.4	101.5	97.6
11	Cost to mine, clean, and load cleaned coal, cents/million B.t.u.[b,c]	8.00	8.17	7.90	7.81	6.66
12	Increase in production cost of cleaned over raw coal, cents/million B.t.u.	0.46	0.63	0.47	0.38	0.56
13	Increase in production cost of cleaned over raw coal, percent/million B.t.u.	6.1	8.4	6.3	5.1	9.2

[a] Assumed.
[b] Assumes cleaned coal moisture, as shipped, is equal to raw coal moisture, as shipped.
[c] On basis of cleaning cost of 10 cents per ton of throughput.
[d] From table 8.

The available data indicate that this is unlikely. In the illustration above it was seen that the cost of production of washed coal is 12.3 percent greater per heat unit than the cost of producing unwashed coal. It follows that an increase of combustion efficiency of 12.3 percent in the utilization of the washed coal would be required to make it comparable in cost to the raw coal per unit of heat actually obtained.

Such an increase in efficiency is out of reason. With an assumed raw coal content of 15 percent, the reduction in ash for this example would be less than five percentage figures, which could hardly be expected to increase efficiency as much as three percent.[44] As a matter of fact, laboratory combustion tests comparing 16 of the cleaned coals prepared in the present investigation[45] with the corresponding raw coals in the same domestic stoker-boiler unit, under standardized conditions and with care to avoid segregation in the handling of both grades of coal, yielded an average efficiency higher by only 1.1 percentage figures (less than two percent gain in efficiency) for the washed coals.[46] For these tests, the average reduction in ash content associated with the washing was 6.2 percentage figures (computed from table 5).

A more important but less easily measured gain in efficiency in the use of the washed coal may be attributed to its increased uniformity. Of two coals identical in cost, average heating value, and average ash content, it is more economical for most purposes to use that which is maintained more uniform in quality, for then an adjustment of the coal-burning unit suitable for the poorest increment of coal fed will

[44] Hebley, Henry F. Economics of preparing coal for steam generation. Trans. AIME vol. 130, pp. 79–100 (TP 847), 1938; p. 85.
[45] Cleaned coals resulting from runs 51, 52, 61, 62, 71, 72, 81, 82, 91, 101, 111, 121, 131, 141, 151, and 161.
[46] Helfinstine, Roy J., and Boley, Charles C. Correlation of domestic stoker combustion with laboratory tests and types of fuels. II. Combustion tests and preparation studies of representative Illinois coals. Illinois Geol. Survey RI 120, 62 pp., 1946; computed from Appendix.

COALS, ESTIMATED FROM LABORATORY COAL-CLEANING DATA

72	81	82	91	101	102	111	112	121	122	131	141	151	152	161
2.8	10.6	10.6	9.9	6.1	6.1	11.0	11.0	13.5	13.5	9.8	11.6	8.2	8.2	12 7
150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
6.10	7.77	7.77	7.51	6.08	6.08	7.28	7.28	7.24	7.24	7.14	6.99	6.14	6.14	6.87
75.6	89.9	79.4	91.1	87.4	89.5	86.0	82.3	86.8	91.1	88.3	86.8	85.7	81.1	81.8
212	178	202	176	183	179	186	194	184	176	181	184	187	197	196
62	28	52	26	33	29	36	44	34	26	31	34	37	47	46
41.1	18.7	34.3	17.1	22.0	19.2	24.0	29.6	22.9	17.1	20.8	22.9	24.5	31.5	30 4
6 8	10 7	13 9	11 5	3 3	3 0	11.7	12 9	5 8	6 2	6.1	5 6	0.8	1.9	5 0
80.7	99.6	90.4	101.5	90.3	92.2	96.0	92.9	91.8	96.8	93.7	91.7	86.4	82.6	85.9
8.06	8.33	9.17	7.88	7.18	7.03	8.09	8.36	8.42	7.98	8.13	8.14	7.58	7.93	8.53
1.96	0.56	1.40	0.37	1.10	0.95	0.81	1.08	1.18	0.74	0.99	1.15	1.44	1.79	1.66
32.1	7.2	18 0	4 9	18 1	15 6	11 1	14 8	16 3	10 2	13.9	16.5	23.5	29.2	24.2

be much closer to being suitable for every other increment than would be true with the more widely varying coal.[47]

However, even allowing for the additional advantages of increased uniformity, the total gain in combustion efficiency appears to fall far short of that necessary to balance increased production cost of washed coal.

REDUCTION IN FREIGHT CHARGES

On the other hand, there is a direct saving to the coal consumer in reduced freight charges on washed coal, per unit of heat. The saving in freight due to the transportation of less inert material becomes increasingly important as the freight rate increases. Eventually a point is reached where savings in freight will compensate for cost of cleaning, and beyond which cleaned coal is cheaper than raw coal, in addition to its other points of superiority.

The freight rate necessary to bring total cost of washed coal (mining plus washing plus freight) down to total cost of raw coal (mining plus freight) for equal heating value is a function of weight yield in the washing process and of percentage increase in heating value, neglecting factors of local pricing policy and dealers' commissions. Figure 14 is a family of curves illustrating the freight rates at which total costs of washed and raw coal, on a heating value basis, come into balance for the assumed set of conditions of $1.50 per ton for cost of mining and 10 cents per ton for cost of washing. In the illustration used above, weight yield was 90 percent and percentage increase in heating value was 5.6, for which a freight rate of $3.50 would be necessary.

Figure 14 makes clear the importance of weight yield in the economics of coal washing. For example, at an increase in heating value of eight percent in the washed coal, a 90-percent yield permits balanced delivered

[47] "The consumer stands to gain more from a uniformly maintained standard quality than from any other single factor when considering the benefits of clean coal versus raw coal." Morrow, J. B., and Davis, D. H. The economics of coal preparation. Chapter I, pp. 1–30, of Coal preparation, David R. Mitchell, Editor; AIME, 729 pp., 1943; p. 26.

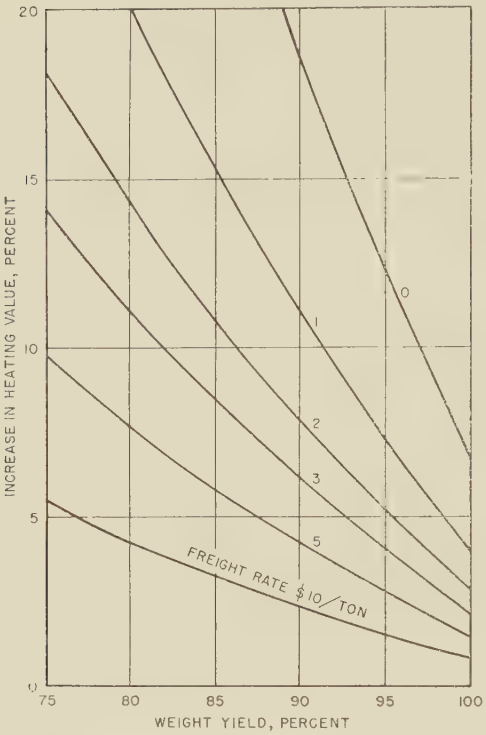

FIG. 14.—Freight rates necessary to equate delivered costs of raw and of washed coal, per B.t.u., for known weight yields and heating value increases.

costs on a B.t.u. basis between raw and washed coal at a freight rate of less than $2.00 per ton under the assumed mining and washing cost conditions. If yield drops to 85 percent, production costs of washed coal are sufficiently higher that a freight rate of over $3.00 per ton is necessary to balance delivered costs.

For the coals washed in the present investigation, column 4 of table 33 gives the freight rates at which delivered costs are balanced. Compared in this fashion, the normally clean coals (no. 10 and 15) appear to be at a disadvantage because washing has improved them less, percentagewise, than it has the dirtier coals.

The length of haul corresponding to any stated freight rate is not definite, but Harrington, Parry, and Koth, in a study of the economics of drying coal, deduced that the freight rate for bituminous slack coal in parts of the country could be roughly estimated (in 1941) as 26.1 cents per ton times the four-tenths power of the haul in miles.[48] Based on this formula, column 5 of table 33 gives the estimated length of haul corresponding to the balanced-cost freight rates for the coals washed in the present investigation. For comparison, the estimated length of haul corresponding to the $3.50 freight rate for the illustrative example previously used is 657 miles.

It seems that the economies in providing a given amount of heat with a washed coal as compared with a raw coal are not large and usually are non-existent, if no account is taken of time spent tending the coal-burning unit. Of course, the inconvenience associated with burning a very high-ash coal may be so great that such a coal is practically unsalable, whereas a relatively small amount of washing will produce from it a coal finding a ready market at a satisfactory price. Under such circumstances washing may be highly profitable. The fact remains that a consumer who places zero value upon his time in caring for his heating plant could get cheaper heat from the unwashed coal. He might in exceptional cases have to provide himself with a larger combustion chamber to meet his demands for heat, although standard equipment will perform remarkably well with high-ash coal if given frequent and proper attention.

Thus the major explanation for the popularity of washed coal must rest with its increased "use value," whereby the time and trouble involved in using coal are reduced. The domestic coal consuming public is willing and anxious to pay relatively high premiums for increased personal convenience and improved performance. Washed coals are much more attractive domestic fuels than unwashed, from nearly every standpoint other than that of cost of heat.

CONVENIENCE

Quantity of ash is of outstanding importance insofar as convenience to the householder is concerned, for all ash must be

[48] Harrington, L. C., Parry, V. F., and Koth, Arthur. Technical and economic study of drying lignite and subbituminous coal by the Fleissner process. U. S. Bur. Mines TP 633, 84 pp., 1942; p. 76.

TABLE 33.—FREIGHT RATES AND ESTIMATED LENGTHS OF HAUL REQUIRED TO BRING WASHED AND RAW COAL COSTS INTO BALANCE, UNDER ASSUMED MINING AND WASHING COSTS[a]

1	2	3	4	5
Run	Weight yield of washed coal, percent[b]	Increase in heating value of washed coal, percent[c]	Freight rate to equalize delivered costs of raw and washed coals, per B.t.u., cents/ton[a]	Estimated length of haul corresponding to freight rate in column 4, miles[d]
51	90.9	10.6	95.3	25
52	88.8	10.8	127.8	53
61	92.6	8.4	123.8	49
62	95.2	6.6	122.7	48
71	92.1	6.0	250.0	284
72	75.6	6.8	761.8	4603
81	89.9	10.7	111.7	38
82	79.4	13.9	224.1	216
91	91.1	11.5	76.1	15
101	87.4	3.3	850.0	6053
102	89.5	3.0	816.7	5477
111	86.0	11.7	157.7	90
112	82.3	12.9	191.1	145
121	86.8	5.8	436.2	1142
122	91.1	6.2	269.4	342
131	88.3	6.1	358.2	698
141	86.8	5.6	457.1	1284
151	85.7	0.8	4475.0	384900
152	81.1	1.9	2323.7	74790
161	81.8	5.0	770.0	4727

[a] Assumed mining cost, $1.50 per ton; assumed washing cost, $0.10 per ton.
[b] From table 5.
[c] From table 8.
[d] Based on freight rate equal to 26.1 x (haul in miles)$^{0.4}$.

handled twice—into the combustion chamber, and out of the ashpit. The increase in convenience due to washing may be illustrated by reference to the previously mentioned 15-percent ash coal, washed to produce a 90-percent weight yield and a 95-percent B.t.u. recovery. Assuming constancy of moisture- and ash-free heating value, the washed coal will have an ash content of 10.3 percent. It is easy to show that the weight of ash which would have to be removed from a coal-burning unit for comparable delivery of heat would be approximately 54 percent greater using the raw than using the washed coal, assuming equal combustion efficiency.

Furthermore, the weight of raw coal into the coal-burning unit must be greater than the weight of washed coal by the same absolute amount; strictly speaking, the increase is about 15 percent more, since the non-combustible material associated with coal usually loses weight upon combustion. Percentagewise, the increase in coal handled before firing is not so striking, approximately 5.5 percent, but its burden on the householder is actually greater.

Other factors of convenience include clinkering characteristics, cleanliness, probability of interruption of service, and odors. Although extensive quantitative data permitting direct comparisons are lacking, a consideration of these factors by Boley and Helfinstine[49] in connection with combustion tests comparing 14 of the washed coals prepared in the present investigation[50] with the corresponding raw coals shows that washed coals are to be preferred for stoker firing in every case.

[49] Boley, Charles C. and Helfinstine, Roy J. Effects of cleaning upon the factors of suitability of stoker coal. Seventh Conference in Coal Utilization, Univ. of Ill., 1946 (1947).

[50] Cleaned coals resulting from runs 51, 52, 61, 62, 71, 81, 91, 101, 111, 121, 131, 141, 151, 161.

Table 34.—Effect of Washing on Maintenance of Desired Temperature Range[a]
(Averages for 14 pairs of coals)

	Raw	Washed
Average uniformity, percentage variation[b]	11.4	7.6
Pickup, thousands of B.t.u. per hour[c]	37.5	40.8
Responsiveness, thousands of B.t.u. per hour[d]	16.1	24.0

[a] As reported in Boley and Helfinstine, op. cit.
[b] Average percentage variation of rate of heat release from the average rate of heat release, during time intervals of arbitrarily selected length. A high number indicates a coal of widely varying rate of heat release.
[c] Average rate of heat release during the first five minutes of stoker operation following a 45-minute "off" period.
[d] Average rate of heat release during the first 30 minutes of stoker operation following a 50-hour hold-fire period.

Performance

As a general rule, washed coals are also capable of distinctly higher levels of performance from the standpoint of maintaining a desired temperature range in the home. Further comparisons of domestic stoker data by Boley and Helfinstine[49] indicate that washed Illinois coals burn with greater uniformity and are more responsive to demand for heat than unwashed coals from the same sources. Table 34 summarizes the pertinent data.

Summary of Advantages of Washed Stoker Coal to Domestic Consumers

Washed coal for domestic stoker use is improved in practically every measurable way. From the standpoint of convenience, less coal and much less ash need be handled; clinkering characteristics are improved; dust raised in coal handling is reduced; probability of interruption of service is reduced; and disagreeable odors are reduced. From the standpoint of performance, uniformity of burning is increased, and responsiveness to demand for heat is increased.

In all but exceptional cases, these advantages involve an increase in the cost of heat, which is, however, usually considered by coal consumers to be well repaid.

Competitive Position

The very willingness to pay for convenience constitutes a major reason why coal's competitive position relative to the fluid fuels—oil and gas—is being weakened, especially in the middle and higher income sectors of population. There is no denying that the fluid fuels are able to supply a degree of convenience and performance not yet approached by coal, usually at a certain additional cost. In some localities the absolute amount of this additional cost is not large, on a yearly basis. Modern small low-heat-loss houses will make it less. The margin available for coal preparation is still less, for few people outside of the coal industry have such loyalty to coal that they will long continue to pay nearly as much for it as for the more convenient fluid fuels. Furthermore, increasing labor costs will penalize coal more than oil or gas, per unit of heat, because wages constitute a much larger percentage of the total value of coal produced than they do of the total value of oil and gas produced.[51]

Coal is thus crowded between inevitably higher cost of production, if the demand for higher quality is to be met, and increased severity of competition from the fluid fuels owing to their greater convenience. It does not seem too early for the coal industry to begin studying the effects on its economy which might be caused by the loss of a substantial proportion of the tonnage now being used for domestic purposes.

[51] Wages paid, 1945: Bituminous coal (including semi-anthracite, lignite, and peat), $1,014,404,000
Crude petroleum and natural gas (including natural gasoline), $464,282,000
(Source: Supplement to National Coal Association Bulletin for June 14, 1947, quoting Social Security Board.)
Value at mines or wells, 1945:
Bituminous coal, $1,774,080,000
Crude petroleum and natural gas, $2,407,226,000
(Source: Minerals Yearbook, 1945.)
Percentage of wages to total value at point of production, 1945:
Bituminous coal, 57.2
Crude petroleum and natural gas, 19.3
Data on the refining of crude petroleum are not included, but it is reasonable to assume that wages in that industry make up no more and probably appreciably less than 19.3 percent of the increase in value of its product.

The Concentrating Table as a Cleaning Device

The economic merits of the concentrating table as a coal-cleaning device may be briefly examined by considering: (1) Size of coal to which it is adapted; (2) capacity per unit of floor space; (3) costs of installation and operation; and (4) capability of the table as a coal cleaner.

SIZE OF COAL TO WHICH THE TABLE IS ADAPTED

Because of the shallow bed carried on the table, the size of particle which can be effectively treated is relatively small. Tables are especially well adapted to the cleaning of coal as sized for metallurgical coking, in the general size range of ¼-in. to 0 or 5/16-in. to 0. Properly riffled and operated, they also do excellent work on coal in the domestic-stoker size range, normally considered to have a maximum size of 1 in. or 1¼ in. Tables are said to be in operation with a feed as large as 3-in. to 2-in., but this is exceptional. It is generally agreed that tables do their best work in treating sizes from ¼-in. down.

Although much single-screened coal is fed to tables with satisfactory overall results, the cleaning effected on the dust below 48-mesh is slight and below 100-mesh is nearly nonexistent.[52] Where water is recirculated, a partially counter-balancing advantage of retaining the dust in the feed results from the building up of the apparent specific gravity of the recirculated wash water.[53] Weight of opinion seems to favor removal of the dust if a very clean product is desired, however.

CAPACITY PER UNIT OF FLOOR SPACE

Full-size coal-washing tables have dimensions approximately double those of the table used in the present investigation (figure 1). To give a minimum of room for operation and maintenance, each table in a battery requires a space at least 24 ft. by 12 ft., or roughly 300 square feet.

Capacity per hour is sometimes quoted as high as 25 tons, varying widely with size of feed and difficulty of cleaning. Feeding at this heavy rate, although perfectly possible, usually results in a poorer performance. More customary rates are 6 to 8 tons per hour for a 5/16-in. to 0 feed, and 10 to 15 tons per hour with larger coal.

Considering the range from 6 to 15 tons per hour, it appears that something of the order of 20 to 50 square feet of floor space is required as a minimum per ton-per-hour capacity, exclusive of all auxiliary materials handling equipment.

Coal-washing equipment of the jig type usually has much larger capacity per square foot of floor space, as also does modern launder type equipment.

COST OF INSTALLATION AND OPERATION

It is doubtful if any other type of cleaning equipment can be purchased and installed as inexpensively as the concentrating table, for plants desiring relatively low capacity (up to 25 tons per hour). For larger capacities, the advantage of high-capacity-per-unit machines becomes more important, and jigs or launders are favored.

The major cost of operation is labor for attendance, although since one man can easily attend 30 tables, labor cost per ton in a large installation is low. Water is the only other significant cost; power consumed is usually well under one horsepower per table, and lost time practically never exceeds one percent.[54]

CAPABILITY OF THE TABLE AS A COAL CLEANER

Within the range of size and capacity to which it is best suited, concentrating tables are regarded as the most efficient and practical coal cleaning device now available.[55] The separation at any time can be easily seen, and with little experience an operator learns how to secure and maintain visually good separation.

The table does not lend itself to increased

[52] Gandrud, B. W. Op. cit., p. 453.
[53] Stone, S. A. Letter to the author, Feb. 12, 1942.
[54] Taggart, Arthur F. Op. cit. pp. 761–2.
[55] Gandrud, B. W. Op. cit., pp. 454–5.

capacity by increases in size, as do many other types of cleaning equipment. It remains a low-capacity-per-unit machine, although capable of excellent performance.

GENERAL

The concentrating table is a low-capacity coal-cleaning device of high efficiency when properly operated with small sizes of coal. It is inexpensive to purchase and install, simple to operate and maintain. Its separation takes place in full view, simplifying close adjustment of the table to the feed and permitting speedy recognition of changes in conditions. A middling product can readily be made.

SUMMARY

Provision was made for washing stoker-size coal by means of a laboratory-size concentrating table (deck dimensions approximately one-half those of a standard coal-washing table), with infinitely variable control of six operating variables over wide ranges during table operation. The six operating variables placed under control were rate of coal feed, rate of introduction of wash water, length of table stroke, frequency of table stroke, transverse table slope, and longitudinal table slope. To permit maximum economy in coal and time in adjusting the table to optimum separation as judged visually, a recirculation system was provided whereby material separated in the normal manner on the table was recombined, freed of all but surface water, and returned to the feed box of the table for repeated separation.

With the equipment complete, twelve Illinois coals, from most commercially important mining districts and coal beds of the state, were subjected to a total of twenty washing tests; and in addition five tests were made by retabling previously tabled coal and two tests were made by retabling material rejected in previous tablings. All coals were sized in the laboratory to a common size range of ½-inch by 8 mesh.

Complete chemical data were obtained for each raw coal and for each cleaned coal, and the percentages of ash and of sulfur were obtained for all rejects and other products necessary for material balances.

A method which was felt to be superior to other methods in common use was devised for the washability analysis of partly dried, high moisture coals. The partly dried coal particles were saturated with water, followed by draining and removal of surface moisture, and fractionation by heavy liquids was so carried out as to avoid completely any exposure of the water-saturated particles to air until they were removed for chemical analysis. Using this method, washability data were obtained for all but one of the test coals.

The methods of partial correlation were used for the analysis of the influence of the operating variables on four measures of performance, independent of variations in nature of coal feed and amount of yield by weight.

Certain phases of the relationship of coal washing to the general economics of coal production were analyzed, and the merits of the concentrating table as a coal washing device were discussed.

CONCLUSIONS

(1) The data indicate fairly conclusively that increased wash water and increased water/coal ratio are conducive to improved results under conditions of constant weight yield, particularly when percentage decrease in mineral matter is accepted as the criterion of improvement.

(2) Less conclusively, the data indicate that increased longitudinal slope and decreased speed of reciprocation tend to promote a cleaner product, for constant weight yield.

(3) Little or no tendency to affect quality of results, independent of weight yield, is shown by the data for rate of coal feed, transverse slope, table stroke, table movement, or composite slope.

(4) Excepting for localities to which the cost of freight is relatively high, washed coal is more expensive to the average domestic coal consumer than raw coal, if no value is placed on convenience of operation or level of performance of the heating plant.

(5) Washed coal is markedly superior to raw coal from the same source for a domestic stoker-fired heating plant from the standpoints of convenience and performance, accounting for its popularity despite its usually higher seasonal cost.

APPENDIX A

The correlation coefficient used in the present treatment of data is the Pearson product-moment correlation coefficient, designed so as quantitatively to characterize the association between two variables. It ranges in magnitude from plus one (indicative of perfect linear relationship between the variables, with large values of one variable associated with large values of the other variable), through zero (indicative of complete independence of the variables), to minus one (indicative of perfect linear relationship, with large values of one variable associated with small values of the other variable). When the variables are expressed in terms of their respective standard deviations, the Pearson coefficient may be defined as "the arithmetic mean of the products of deviations of corresponding values from their respective arithmetic means."[56] Algebraically, this may be expressed

$$(1) \qquad r_{xy} = \frac{\Sigma (x - \bar{x})(y - \bar{y})}{N\sigma_x\sigma_y},$$

where r_{xy} is the coefficient of correlation between variables x and y, $\bar{x}$ and $\bar{y}$ are the arithmetic means of variables x and y respectively, N is the number of pairs of data in x and y, and σ_x and σ_y are the standard deviations of the variables x and y respectively.

Normally, variables are not expressed in terms of their standard deviations, and it is not convenient to compute their deviations from their arithmetic means. For purposes of computation it is usually more convenient to use the expression

$$(2) \qquad r_{xy} = \frac{\Sigma xy - \bar{x}\,\Sigma y}{N\sigma_x\sigma_y}.$$

This may be expressed in words as "the summation of every x multiplied by the corresponding y, diminished by the product of the mean of the x's and the total of the y's, all divided by the product of the number of pairs of data, the standard deviation of the x's, and the standard deviation of the y's." This definition is identical with that given in the first paragraph but is better adapted to computation.

The standard deviation of any collection of numbers is a measure of their dispersion, or "scatter," just as the arithmetic mean (the most common type of average) is a measure of their central tendency. The standard deviation is given by the "root-mean-square" of the deviations of the numbers from their arithmetic mean; that is,

$$(3) \qquad \sigma_x = \left\{\frac{\Sigma (x - \bar{x})^2}{N}\right\}^{1/2},$$

or, expressed more conveniently for computation,

$$(4) \qquad \sigma_x = \left\{\frac{\Sigma x^2 - \bar{x}\,\Sigma x}{N}\right\}^{1/2}.$$

Where several variables and any appreciable number of units of data are involved, it is practically essential to use a calculating machine, preferably of the crank-driven type. As can be seen in the expressions for σ_x and r_{xy}, the sums of the squares of each value of each variable and the sums of each value of each variable multiplied by the corresponding values of each other variable are required. In doing the calculating, it is highly desirable to adopt some systematic scheme to permit cross-checking of the work as it proceeds. An excellent system for doing this is described in full with detailed examples in pages 29 to 35 of Wallace and Snedecor.[57]

APPENDIX B

The partial correlation coefficient between variables x and y, independent of variable z, is the total, or "zero-order," correlation coefficient between x′ and y′, where x′ and y′ are values of x and y predicted on the basis of knowledge of z.[58] It follows from formula (1), Appendix A, that

$$(1) \qquad r_{xy\cdot z} = \frac{\Sigma (x - x')(y - y')}{N\sigma_{x\cdot z}\sigma_{y\cdot z}},$$

where $r_{xy\cdot z}$ is the partial correlation coefficient between variables x and y, independent of variable z, $\sigma_{x\cdot z}$ and $\sigma_{y\cdot z}$ are the standard deviations of the residuals $(x - x')$ and $(y - y')$ respectively, and N is the number of units of data in x, y, and z.

It may be shown[59] that

$$(2) \qquad r_{xy\cdot z} = \frac{r_{xy} - r_{xz}r_{yz}}{\left[(1 - r_{xz}^2)(1 - r_{yz}^2)\right]^{1/2}},$$

which is usually more convenient for computation.

A partial correlation coefficient expressing the relationship between two variables independent of n other variables is said to be of the nth order. Partial correlation coefficients of higher orders may be built up from those of the next lower order.[60] For example, the partial correlation coefficient expressing the relationship between x and y, independent of w and z ($r_{xy\cdot wz}$), is of the second order and may be expressed in terms of three first-order partial correlation coefficients, thus,

$$(3) \qquad r_{xy,wz} = \frac{r_{xy,z} - r_{wx,z}r_{wy,z}}{\left[(1 - r_{wx,z}^2)(1 - r_{wy,z}^2)\right]^{1/2}}.$$

[56] Rietz, Henry Lewis. Mathematical statistics. No. 3 of the Carus Mathematical Monographs, Open Court Pub. Co. (Chicago), 181 pp., 1927; p. 83.

[57] Wallace, H. A., and Snedecor, George W. Op. cit.

[58] Rietz, Henry Lewis. Op. cit., p. 99. Also, Wallace, H. A., and Snedecor, George W. Op. cit., p. 49.

[59] Rietz, Henry Lewis. Op. cit., p. 100.

[60] Rietz, Henry Lewis. Op. cit., p. 101.

Similarly, partial correlation coefficients of the third order, such as were used in the present treatment of data, may be computed from three second-order, or six first-order (allowing for duplication), or ten zero-order correlation coefficients.

Calculations made in this way are very extensive, although fairly simple in form. A more direct method for computing partial correlation coefficients of higher orders rests upon the fact that any correlation coefficient is the geometric mean of the two so-called regression coefficients of the same order. One regression coefficient represents the average change in variable x per unit change in variable y, and the other represents the average change in variable y per unit change in variable x, both independent of as many other variables as indicated by their order. In general, the two regression coefficients are not the same and, except in the case of perfect correlation, are not reciprocals of each other.

A common notation for the zero-order regression coefficient of x on y (average change in x corresponding to unit change in y) is b_{xy}, and for first- and higher-order regression coefficients of x on y is β_{xyz}, where the variables, the effects of which are removed, are shown to the right of the dot. Thus, from the foregoing paragraph,

$$r_{xy} = \sqrt{(\beta_{xy.z})(\beta_{yx.z})}, \tag{4}$$

and also

$$r_{xy.wz} = \sqrt{(\beta_{xy.wz})(\beta_{yx.wz})}. \tag{5}$$

Ingenious schemes have been devised for computing higher-order regression coefficients, which are also needed in working out multiple correlation coefficients.[61] These schemes are intended to systematize the work, to save labor, and to promote accuracy. An outline of the method used to compute one second-order partial regression coefficient, $\beta_{xy.wz}$, appears at the bottom of this page.

In this outline, lines 1, 3, and 7 consist simply of the six zero-order correlation coefficients relating variables w, x, y, and z; and the other lines are self-explanatory in the operations to be performed.

To obtain $\beta_{yx.wz}$, a new outline is set up with the last two columns interchanged in lines 1 and 3, and the same system of calculation is carried out. The second-order partial correlation coefficient, $r_{xy.wz}$, may then be computed by formula (5).

The present treatment of data involves third-order partial correlation coefficients, which require somewhat more extensive calculations, but it is believed that the expansion necessary for this work will be clear if one will carry out the operations indicated above for second-order partial correlation coefficients.

[61] The most careful explanation known to the author of such a scheme appears in Wallace, H. A., and Snedecor, George W., Op. cit., and their procedure has been followed in the computations made for the present treatment of data. Unfortunately, this publication is now out of print.

1	1.0	r_{zw}	r_{zy}	r_{zx}
2	-1.0	$-r_{zw}$	$-r_{zy}$	$-r_{zx}$
3		1.0	r_{wy}	r_{wx}
4		$-r_{zw}^2$	$-r_{zw}r_{zy}$	$-r_{zw}r_{zx}$
5		$1-r_{zw}^2$	$r_{wy}-r_{zw}r_{zy}$	$r_{wx}-r_{zw}r_{zx}$
6		-1.0	$-\dfrac{r_{wy}-r_{zw}r_{zy}}{1-r_{zw}^2} = -\beta_{yw.z}$	$-\dfrac{r_{wx}-r_{zw}r_{zx}}{1-r_{zw}^2} = -\beta_{xw.z}$
7			1.0	r_{yx}
8			$-r_{zy}^2$	$-r_{zy}r_{zx}$
9			$-\beta_{yw.z}(r_{wy}-r_{zw}r_{zy})$	$-\beta_{xw.z}(r_{wx}-r_{zw}r_{zx})$
10			$1-r_{zy}^2-\beta_{yw.z}(r_{wy}-r_{zw}r_{zy})$	$r_{yx}-r_{zy}r_{zx}-\beta_{xw.z}(r_{wx}-r_{zw}r_{zx})$
11			-1.0	$-\dfrac{r_{yx}-r_{zy}r_{zx}-\beta_{xw.z}(r_{wx}-r_{zw}r_{zx})}{1-r_{zy}^2-\beta_{yw.z}(r_{wy}-r_{zw}r_{zy})} = -\beta_{xy.wz}$

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VITA

Charles Chilton Boley was born in La Crosse, Wisconsin, on July 3, 1911. He was graduated from J. Sterling Morton High School, Cicero, Illinois, in 1928, and attended Morton Junior College from September, 1928, to June, 1930. After a period of work in Chicago, during which he attended night school on the Chicago campus of Northwestern University from September, 1932, to June, 1933, he matriculated at the University of Illinois in September, 1933, and received the degree of Bachelor of Science in Mining Engineering *cum laude,* in June, 1935. In July, 1935, he was employed as a Technical Assistant on a part-time basis in the Coal Division of the Illinois State Geological Survey and remained in the employ of the Geological Survey until October, 1946, advancing to the rank of Associate Mining Engineer. During this period, he continued to attend the University of Illinois, receiving the degree of Master of Science in Mining Engineering in February, 1938. In October, 1946, he was employed as Coal Research Engineer by the Natural Resources Research Institute, University of Wyoming. He is a member of the American Institute of Mining and Metallurgical Engineers, the Illinois Mining Institute, the Rocky Mountain Coal Mining Institute, the Illinois State Academy of Science, the Colorado-Wyoming Academy of Science, the American Chemical Society, Sigma Xi, and the American Association for the Advancement of Science. He is a Registered Professional Engineer under the laws of the State of Illinois.

He has collaborated in the following publications:

Boley, Charles C., and McCabe, L. C. Separation and concentration of vitrain, clarain, and fusain in Illinois coal. Trans. Ill. State Acad. Sci. *29* (2), 153-5, Dec. 1936.

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McCabe, Louis C., and Boley, Charles C. Physical properties of coals. Chap. 7 of Chemistry of coal utilization (H. H. Lowry, Ed.), Wiley (New York), 1868 pp., 1945; pp. 310-36.

Helfinstine, Roy J., and Boley, Charles C. Correlation of domestic stoker combustion with laboratory tests and types of fuels. II. Combustion tests and preparation studies of representative Illinois coals. Ill. State Geol. Surv. RI 120, 62 pp., 1946.

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